

## Atmospheric Physics

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# Earth's atmosphere and circulation

# Atmospheric composition

## Composition

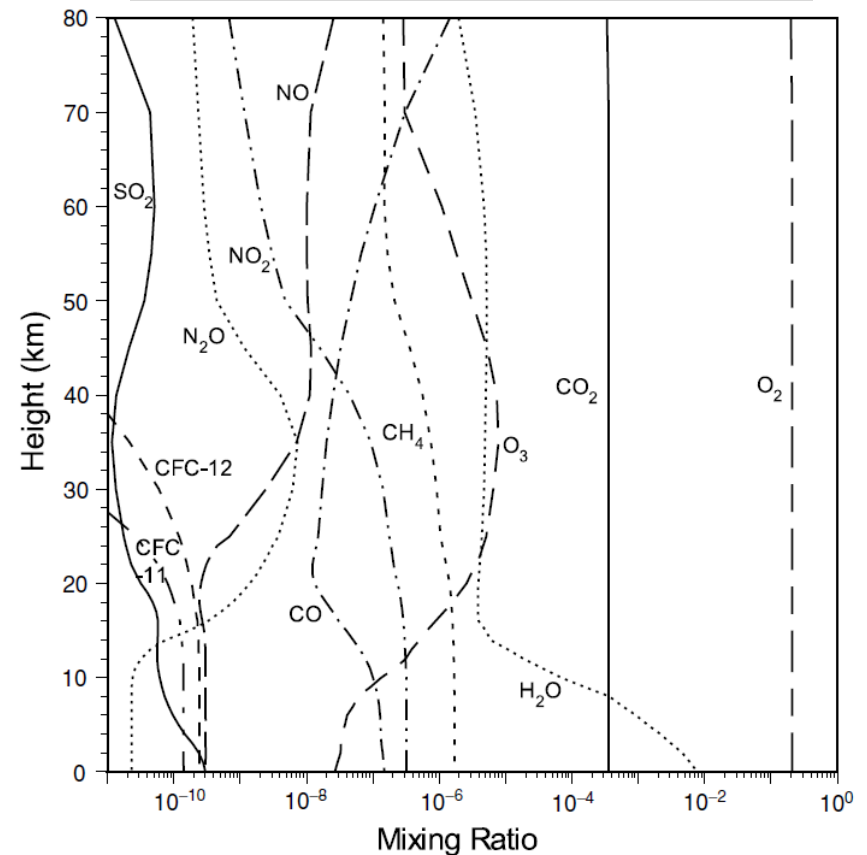
### Gaseous composition of dry air

Constituent	Chemical symbol	Mole percent
Nitrogen	N <sub>2</sub>	78.084
Oxygen	O <sub>2</sub>	20.947
Argon	Ar	0.934
Carbon dioxide	CO <sub>2</sub>	0.0370
Neon	Ne	0.001818
Helium	He	0.000524
Methane	CH <sub>4</sub>	0.00017
Krypton	Kr	0.000114
Hydrogen	H <sub>2</sub>	0.000053
Nitrous oxide	N <sub>2</sub> O	0.000031
Xenon	Xe	0.0000087
Ozone*	O <sub>3</sub>	trace to 0.0008
Carbon monoxide	CO	trace to 0.000025
Sulfur dioxide	SO <sub>2</sub>	trace to 0.00001
Nitrogen dioxide	NO <sub>2</sub>	trace to 0.000002
Ammonia	NH <sub>3</sub>	trace to 0.0000003

\*Low concentrations in troposphere; ozone maximum is found at 30- to 40-km above Earth's surface in the equatorial region.

(After Warneck, 1988; Anderson, 1989; Wayne, 1991.)

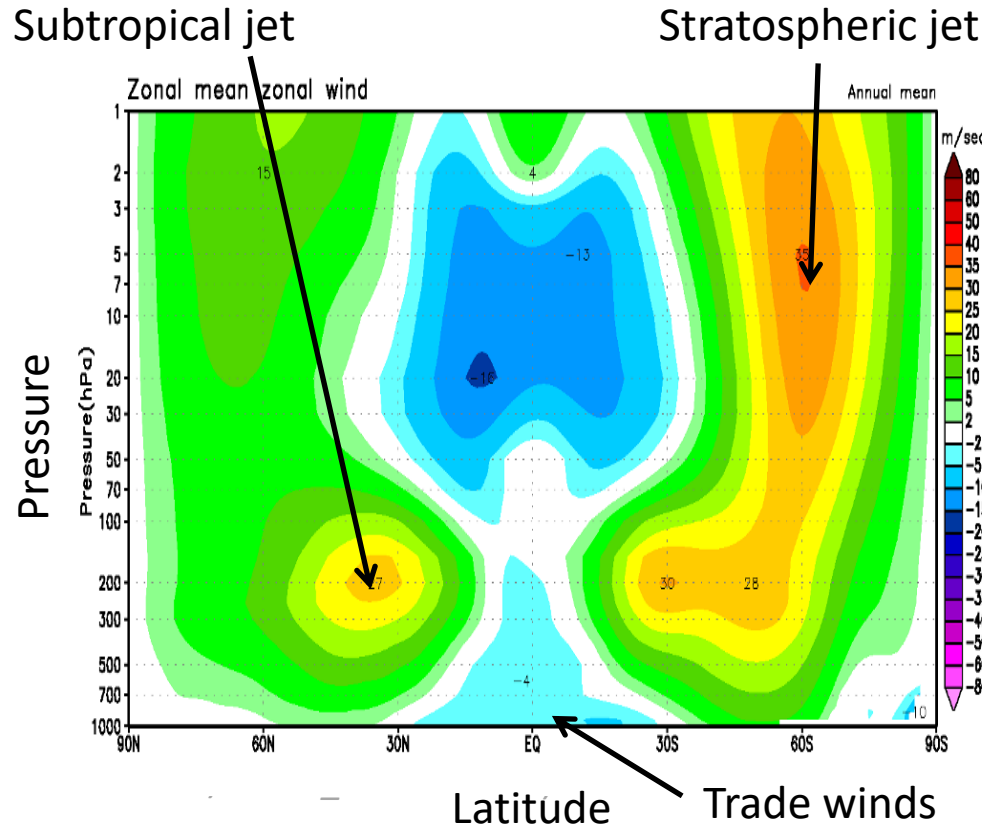
## Vertical distribution of constituents





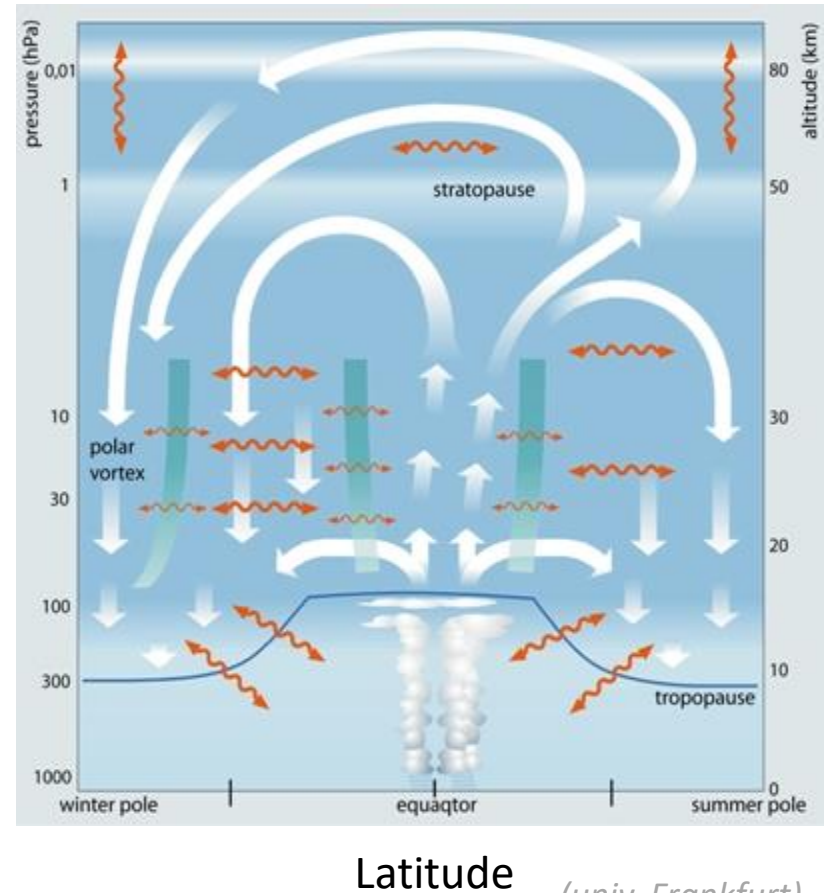
# Large scale circulation

Annual zonal wind  
zonal mean ( $\langle U \rangle$ )



([http://ds.data.jma.go.jp/gmd/jra/atlas/eng/indexe\\_isobar13.htm](http://ds.data.jma.go.jp/gmd/jra/atlas/eng/indexe_isobar13.htm))

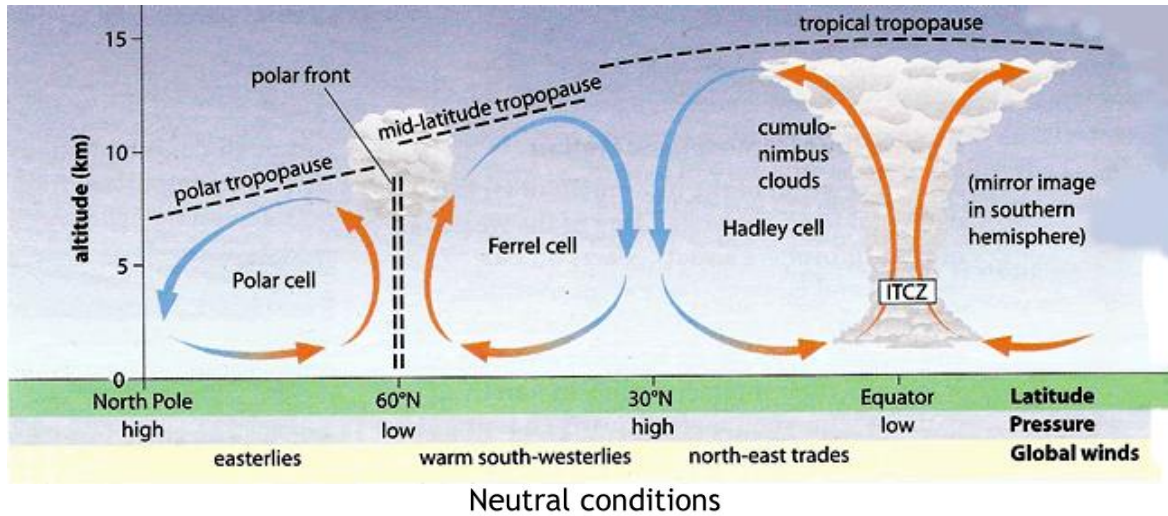
Stratosphere meridional overturning circulation



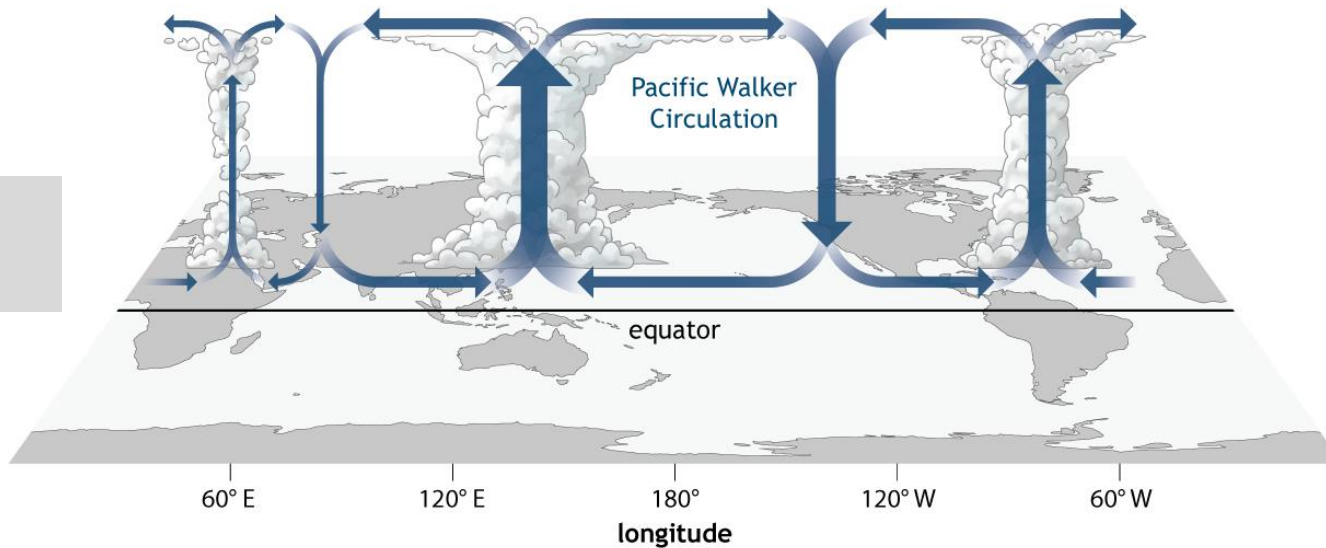
Clear asymmetry between both hemisphere

## Large scale circulation (2)

Meridional/height circulation

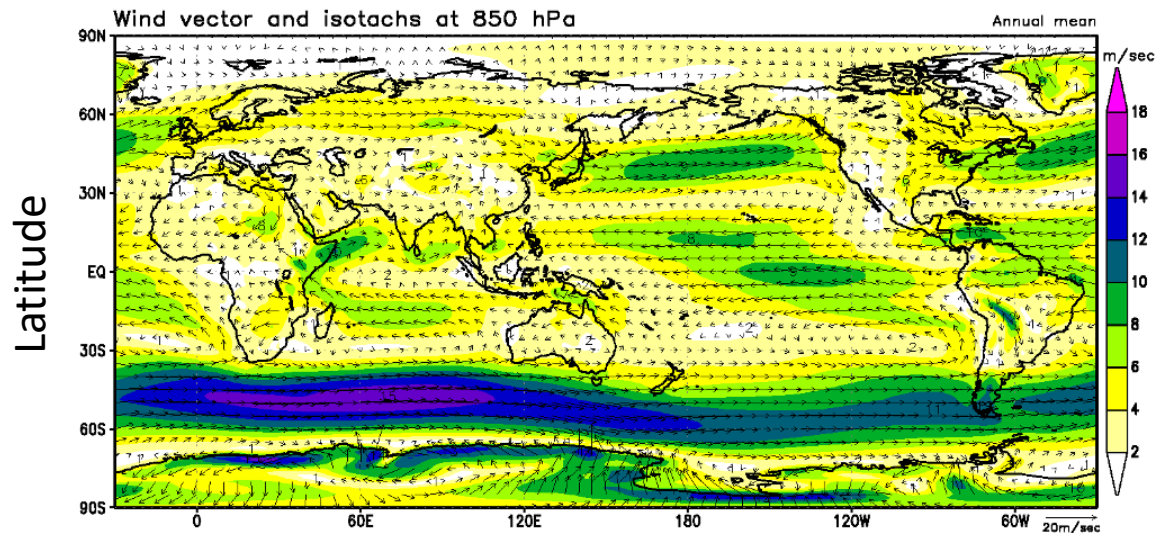


Zonal/height circulation

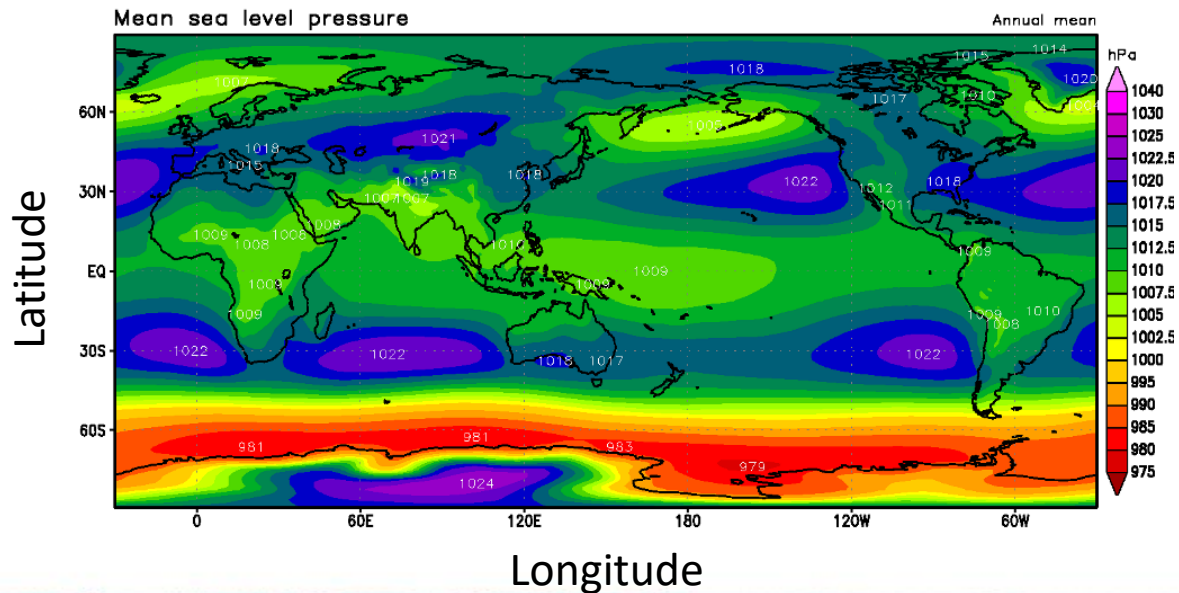


# Regional circulation

Annual mean wind direction and velocity at 850 hPa (~1.5 km)



Annual mean sea level pressure (hPa)

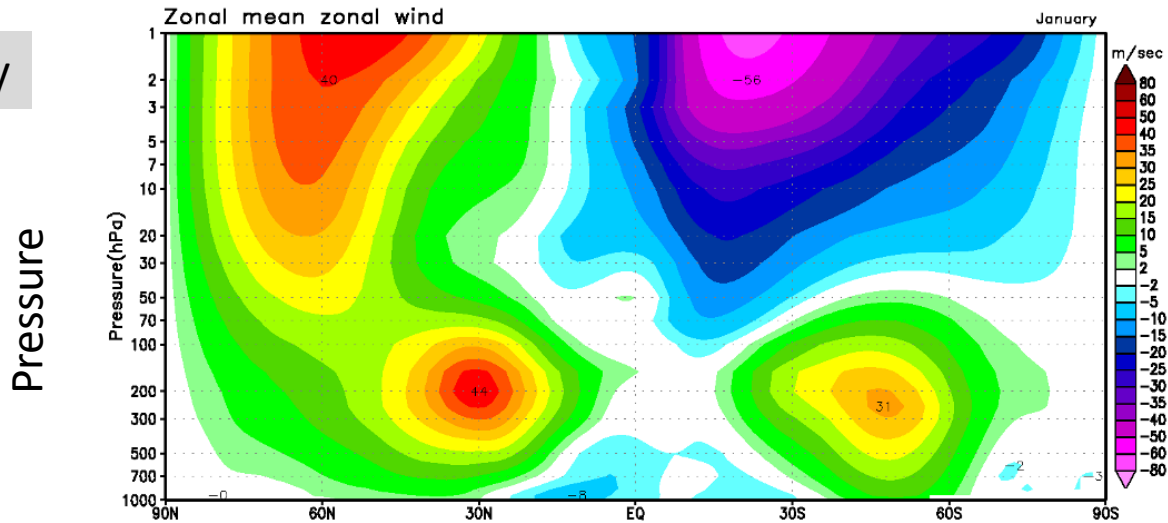


([http://ds.data.jma.go.jp/gm/d/jra/atlas/eng/indexe\\_isobar13.htm](http://ds.data.jma.go.jp/gm/d/jra/atlas/eng/indexe_isobar13.htm))

## Seasonality

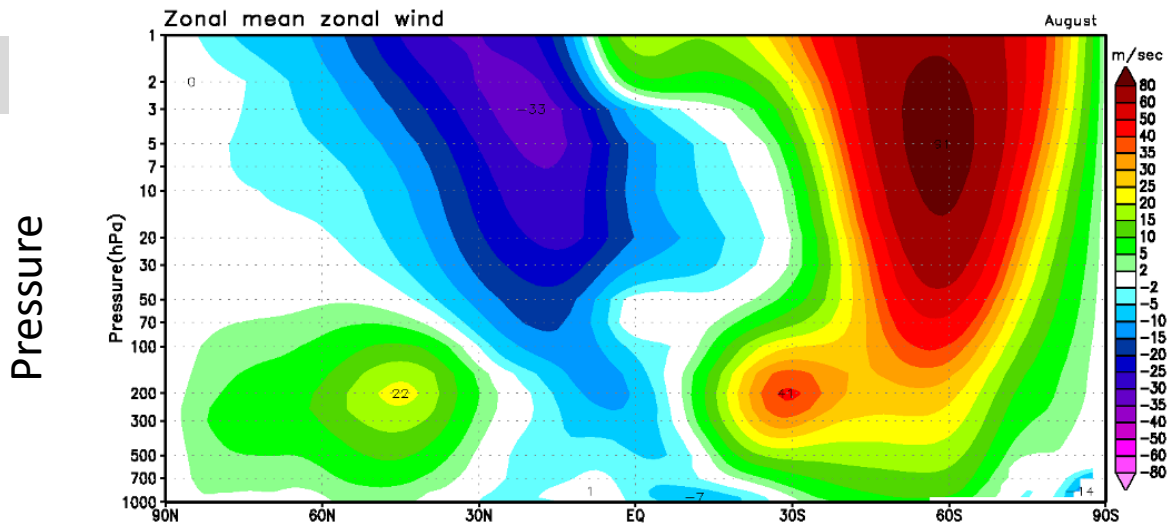
### Mean zonal wind zonal mean

January



Clear asymmetry between both hemispheres at different seasons

August



([http://ds.data.jma.go.jp/gmd/jra/atlas/eng/indexe\\_isobar13.htm](http://ds.data.jma.go.jp/gmd/jra/atlas/eng/indexe_isobar13.htm))

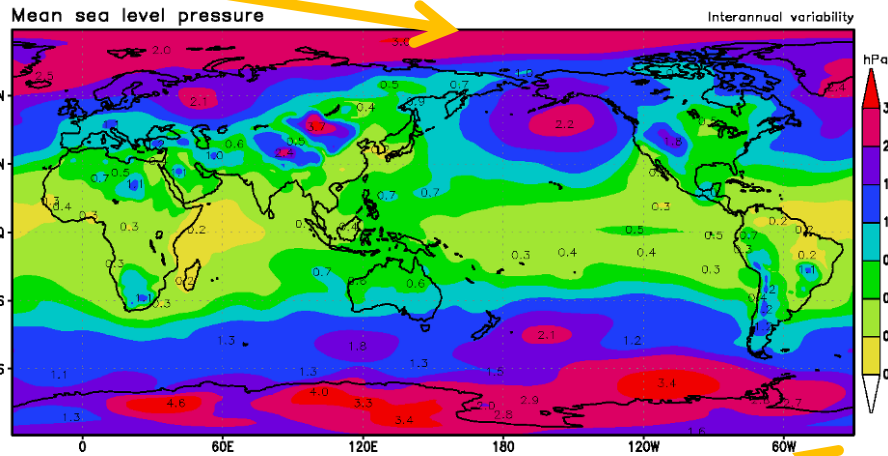


## Variability

AO  
(Arctic Oscillation)

SLP (hPa)

latitude

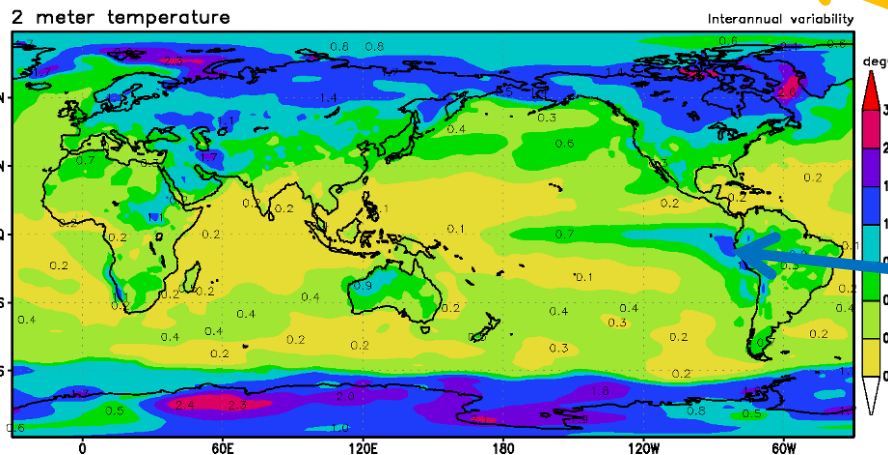


AAO  
(Antarctic  
Oscillation)

ENSO

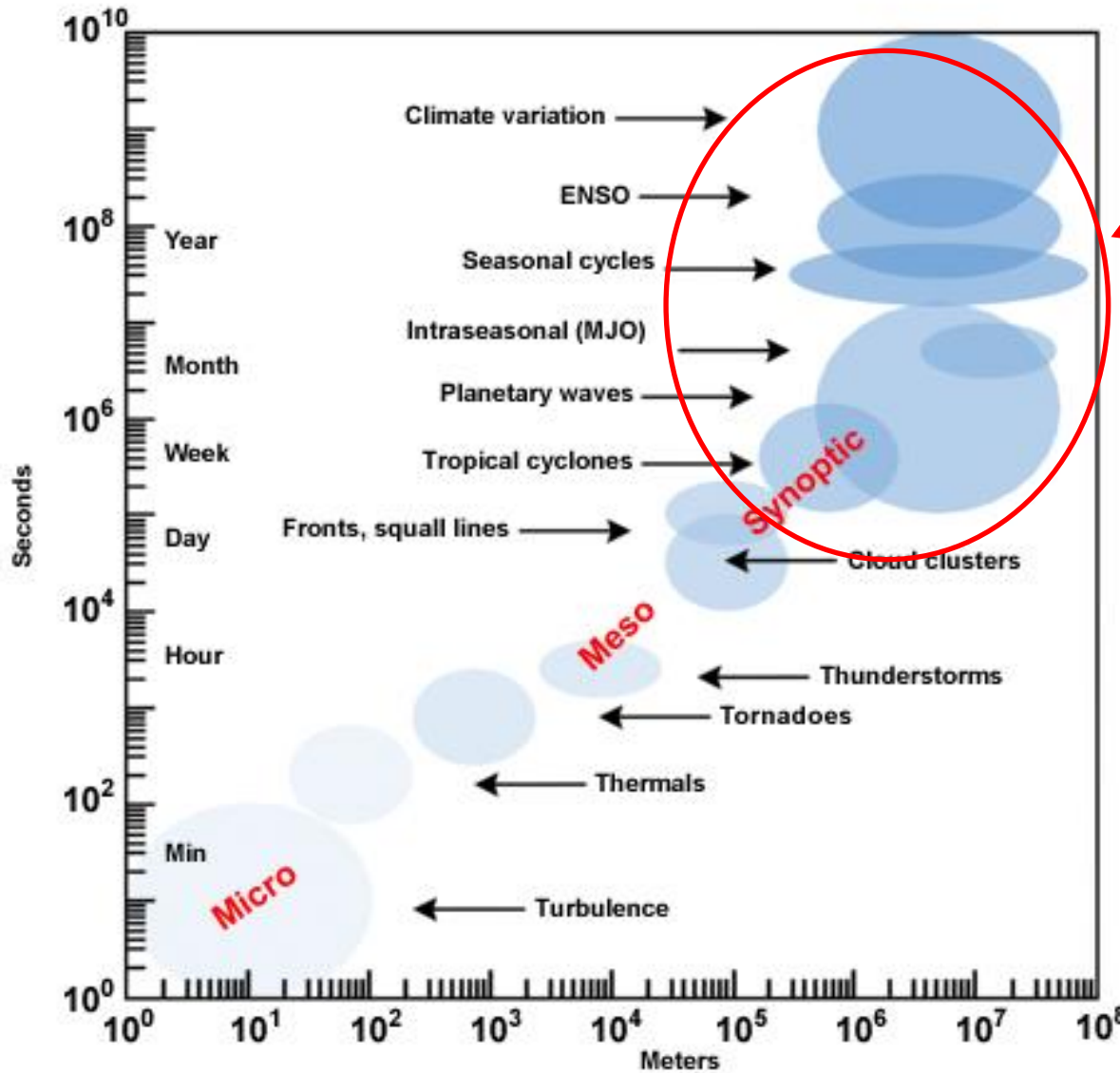
2m Temp

latitude



longitude

## Scales



Focus of the lecture

What drive the atmosphere  
dynamical processes ?

# Outline

I. Radiations

II. Thermodynamical processes

III. Dynamics

IV. Stratosphere

# I. Earth's radiative balance

## Energy source: the Sun

The sun can be considered as a „Black body“:

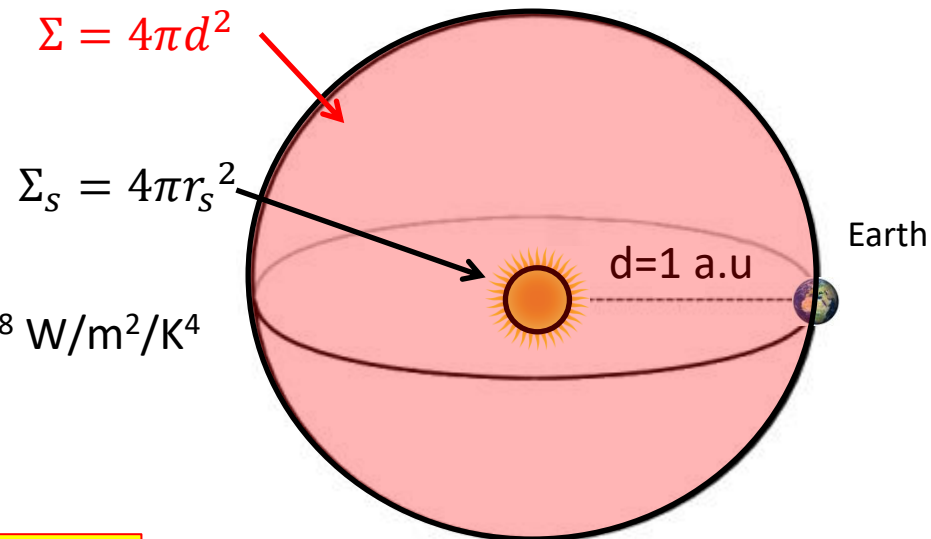
Energy flux (Stefan’s law):  $F_s = \sigma T_s^4$

$T_s$ : Sun temperature

$R_s$ : Sun radius

$= 5.67 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4$

Energy conservation:  $\Sigma_s * F_s = \Sigma * F$



$$S = F_{TOA} = \sigma T_s^4 \left( \frac{r_s}{d} \right)^2 \approx 1360.8 \text{ Wm}^{-2}$$

(TOA: Top of the Atmosphere)

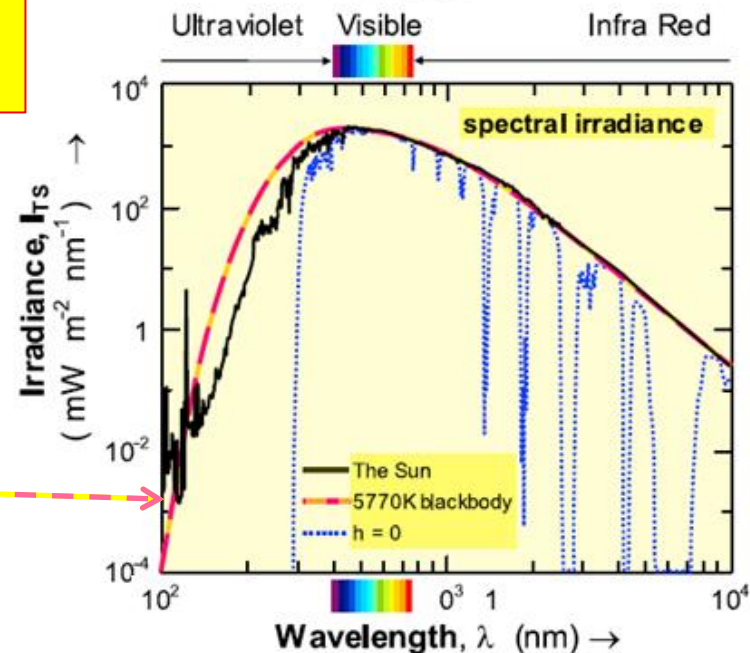
**Solar constant (TSI:  
Total Solar Irradiance)**

Spectral Solar Irradiance (SSI):

$$F(\lambda, T) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k T)} - 1}$$

(w/m<sup>2</sup>/m)

h: Planck constant, c: speed of light, k: Boltzmann constant



Gray et al., 2010

## Earth's radiative balance: model without atmosphere

### Energy balance:

Part of the SW are directly reflected back to space



$$\pi a^2 (S - S\alpha) = 4\pi a^2 \sigma T_{earth}^4$$

$a$ : Earth's radius

$\alpha$ : albedo

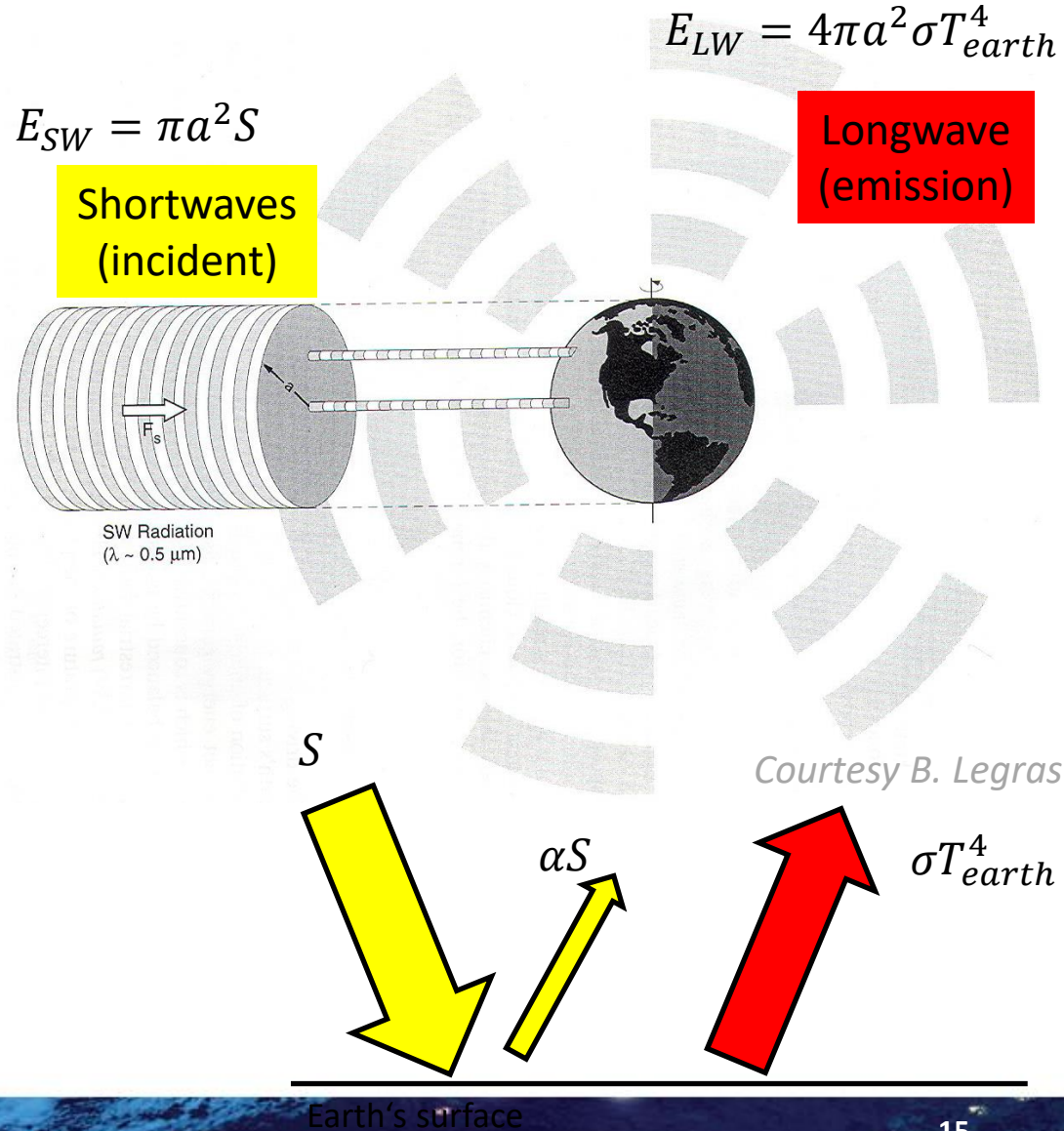


$$\frac{S}{4} (1 - \alpha) = \sigma T_{earth}^4$$



$$\alpha_{earth} = 0.3$$

$$T_{earth} = -18^\circ\text{C}$$



## Earth's radiative balance: single layer model

Atmosphere is transparent to visible

Energy balance surface:  $\pi a^2(S - S\alpha) - 4\pi a^2\sigma T_{surface}^4 + 4\pi a^2\sigma T_{atmosphere}^4 = 0$

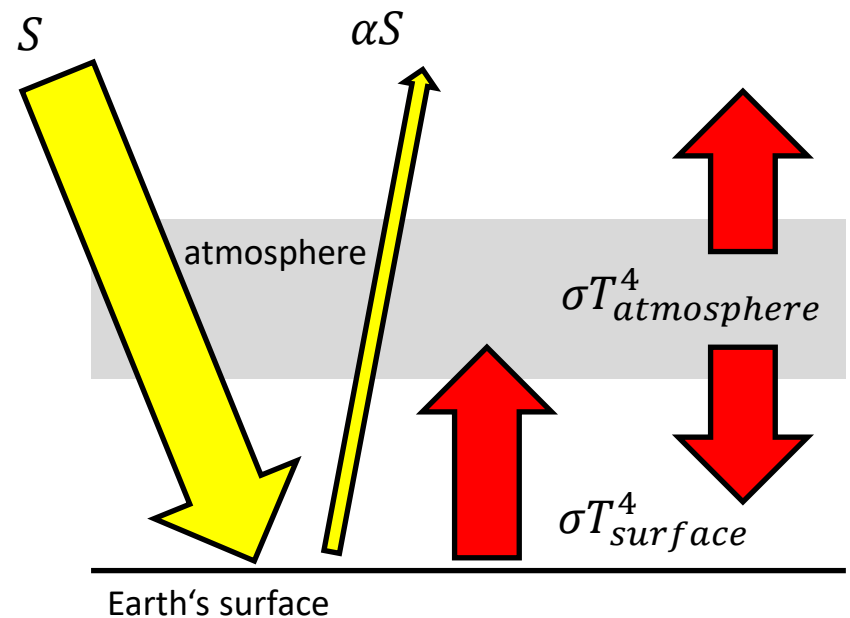
Energy balance atmos:  $4\pi a^2\sigma T_{surface}^4 = 2 * 4\pi a^2\sigma T_{atmosphere}^4$

$\sigma T_{eq}^4$ : (temperature without atmosphere)

$$\frac{S}{4}(1 - \alpha) - \frac{1}{2}\sigma T_{surface}^4 = 0$$

$$T_{surface}^4 = 2T_{eq}^4$$

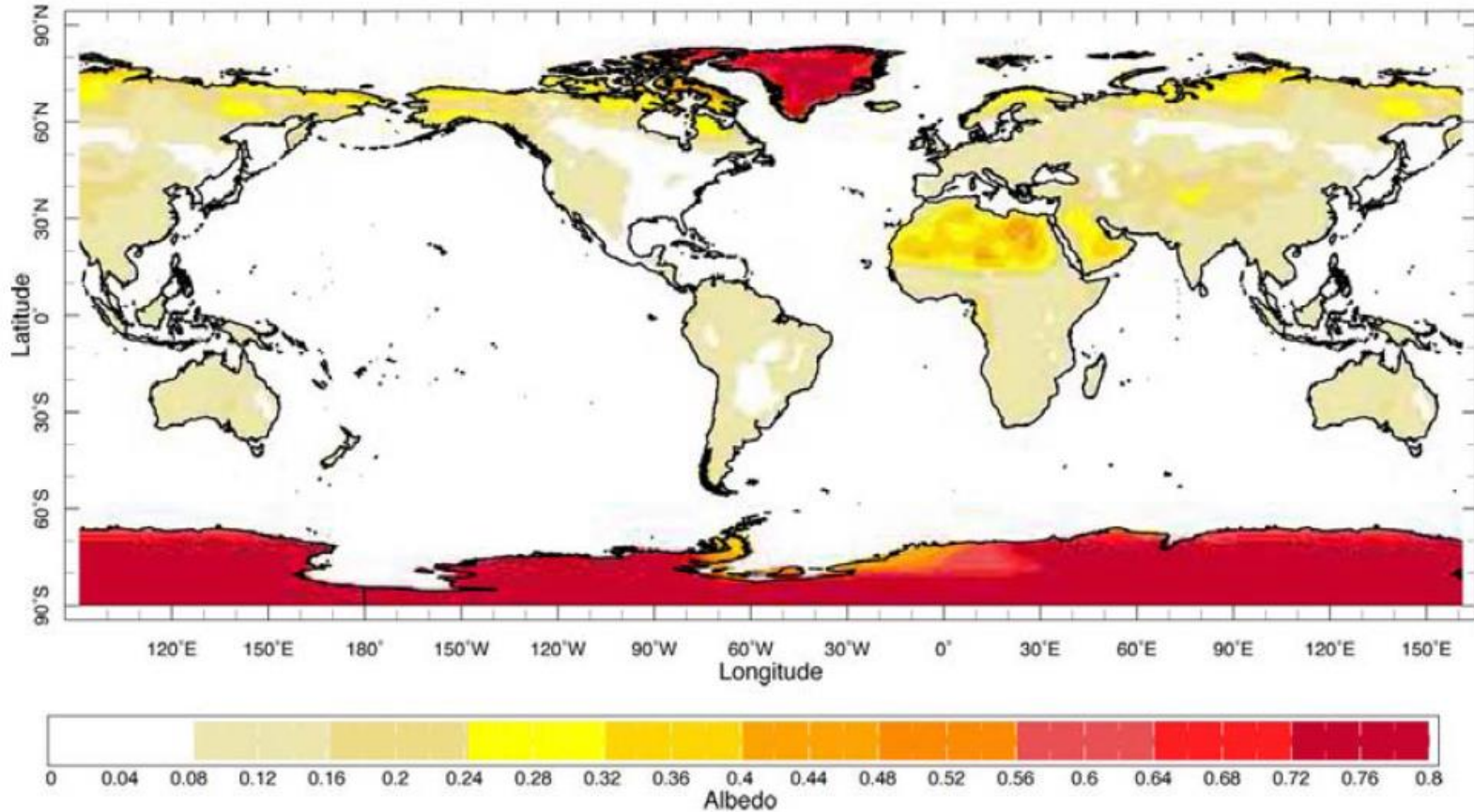
$$T_{surface} = 15^\circ\text{C}$$





## Earth's albedo

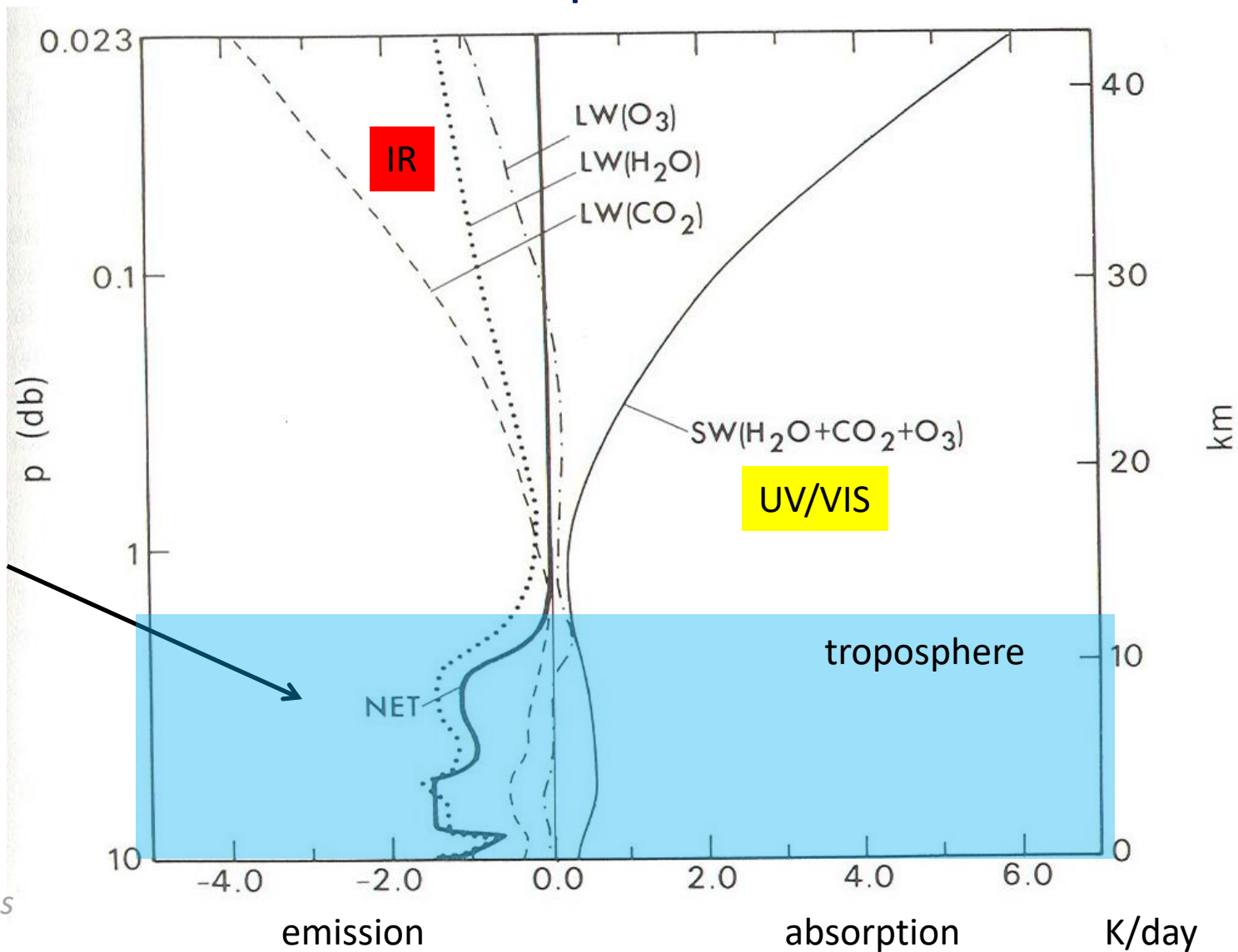
### Surface Albedo



# Earth's radiative balance: vertical profile

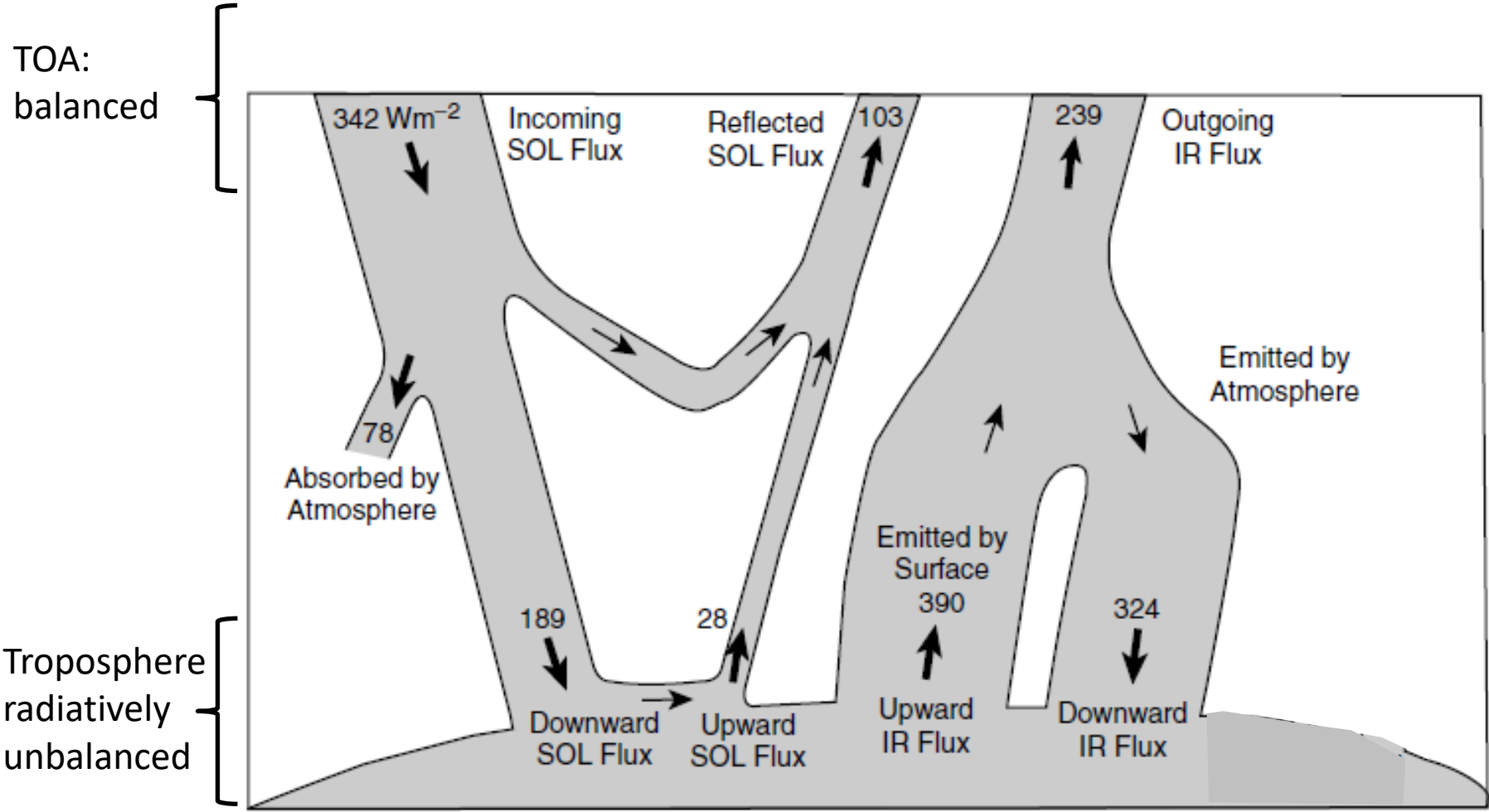
Stratosphere in radiative balance (UV absorption by  $O_3$  and IR emission by  $CO_2$ ).

Troposphere must be balanced.



Courtesy B. Legras

# Earth's energy budget



Liou, 2002

## Earth's radiative balance

UV/VIS

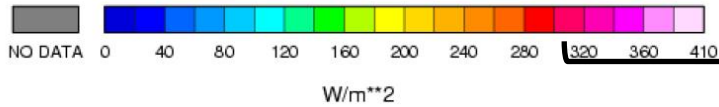
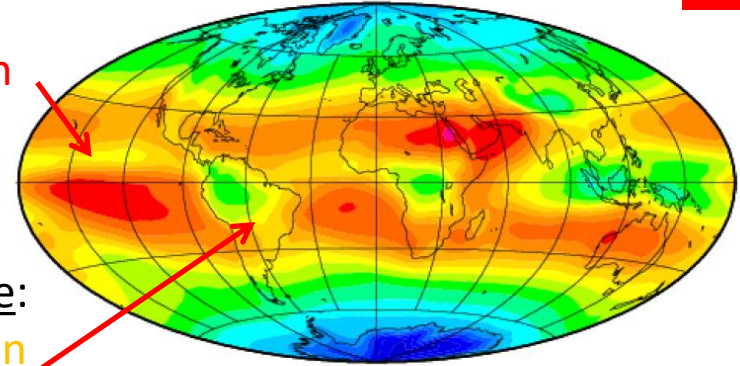
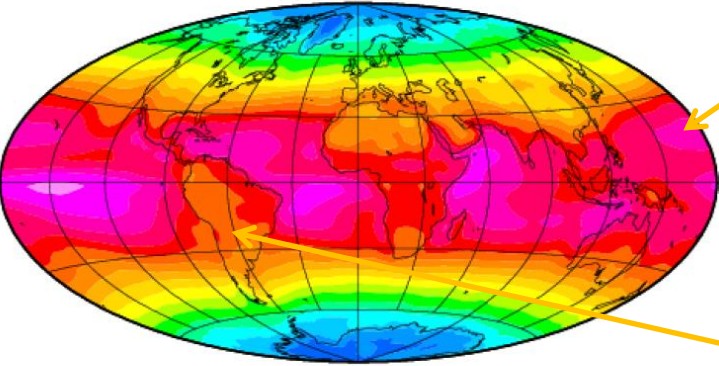
Absorbed Shortwave Radiation  
1985-1986

Ocean: **strong**  
absorption  
and **emission**

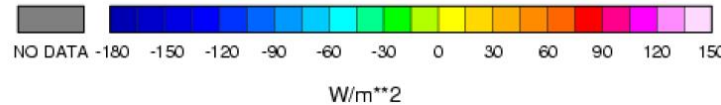
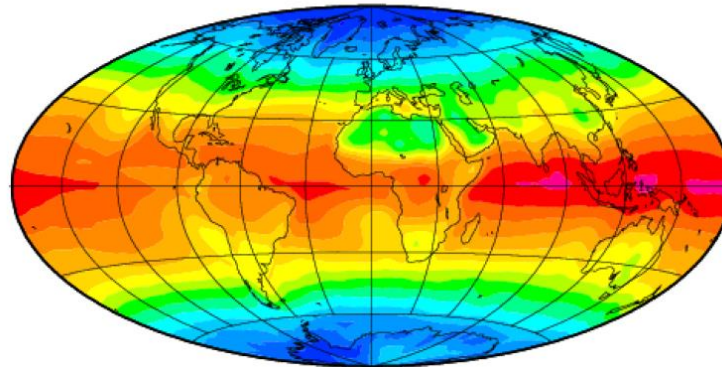
Outgoing Longwave Radiation  
1985-1986

IR

Convective zone:  
**weak** absorption  
and **emission**



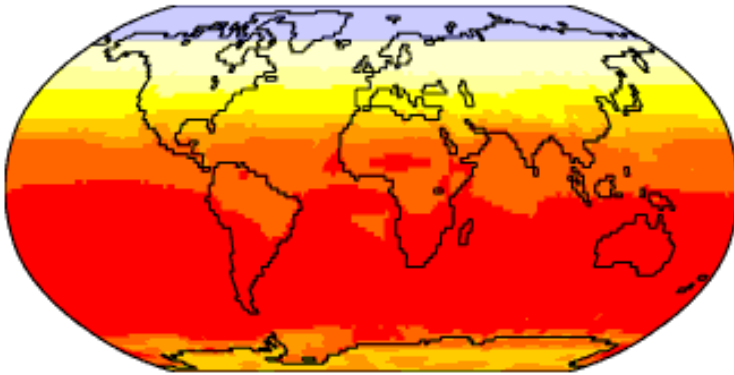
Net Radiation  
1985-1986



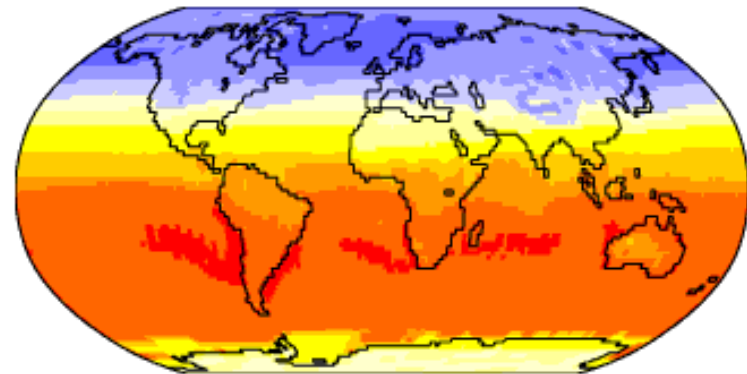
## Earth's radiative balance: seasonality

Short-Wave Radiation

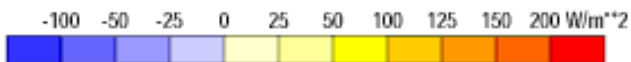
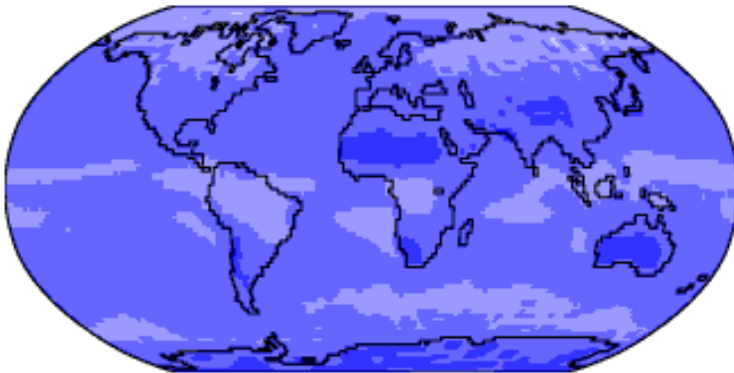
Dec



Net Radiation



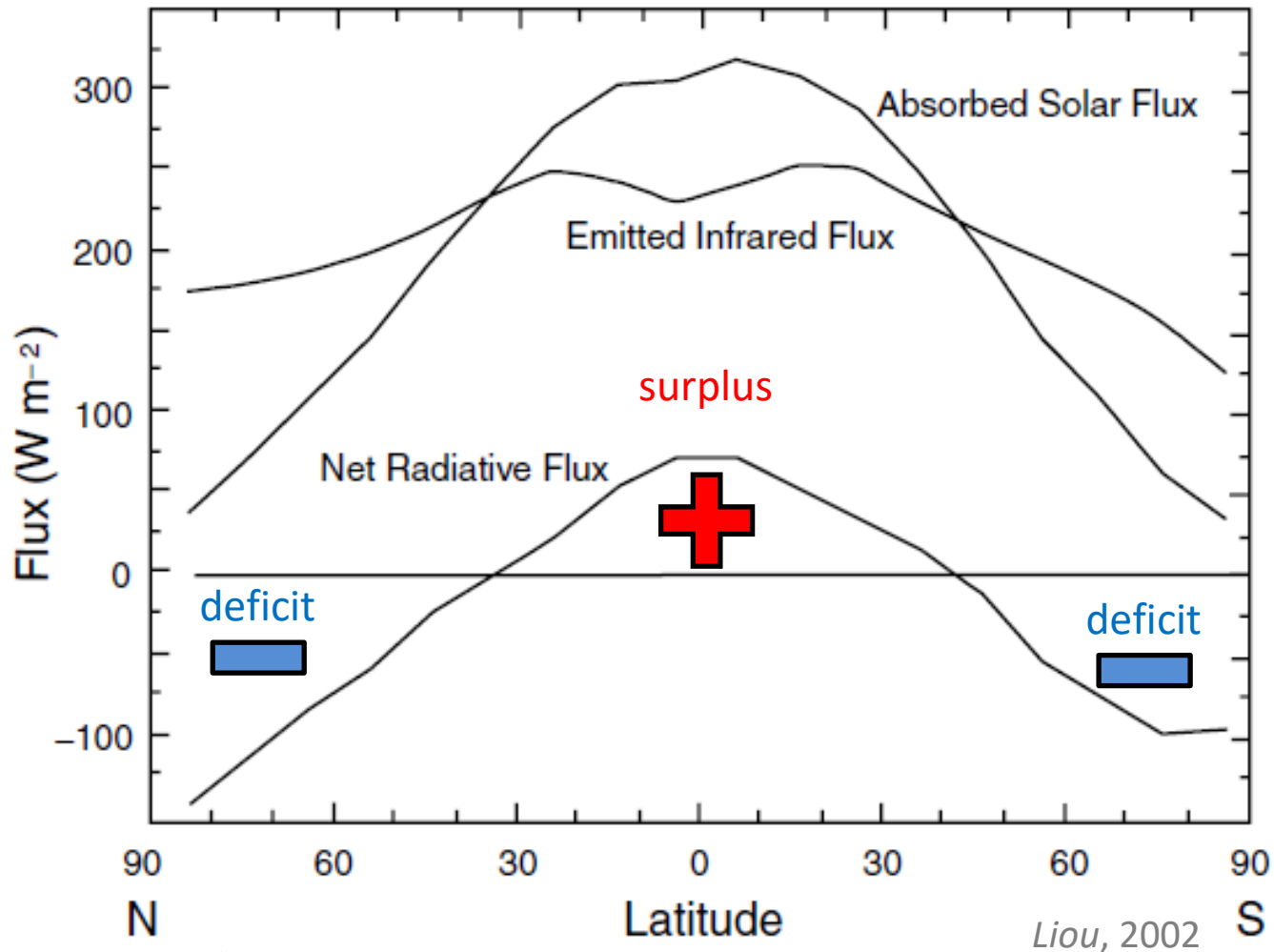
Long-Wave Radiation



Data: NCEP/NCAR Reanalysis Project, 1959-1997 Climatologies  
Animation: Department of Geography, University of Oregon, March 2000

([http://geog.uoregon.edu/envchange/clim\\_animations/](http://geog.uoregon.edu/envchange/clim_animations/))

## Earth's distribution



**Driver of the atmosphere's general circulation**

## II. Thermodynamical processes

## Pressure-altitude equation

For dry air ( $\Leftrightarrow$  no water vapor):

Gas constant: 8.314 J/(K.mol)

Gas law:

$$p = \rho_{air}RT, \text{ with } R = R^*/M_{air}$$

$$\cong 0.21 \times M_{O_2} + 0.78 \times M_{N_2} + 0.01 \times M_{Ar} = 28.9 \text{ g/mol}$$

Hydrostatic equation:

$$dp/dz + \rho g = 0$$

$$dp/p = \frac{-gM_{air}}{R^*T} dz$$

integrate

$$p(z) = p_0 e^{\left(-\int_{z_0}^z \frac{gM_{air}}{R^*T(z)} dz\right)}$$

Under isothermal conditions ( $T_0=255 \text{ K}$ ):  $p(z) = p_0 e^{(-z/H)}$ , with  $H=R^*T_0/(gM_{air})=7.4 \text{ km}$

$H$  (scale height)  $\Leftrightarrow$  pressure divided by 2 every 5 km ( $H \ln(2)=5 \text{ km}$ )



## Standard Atmosphere Approximation (ICAO)

Ground conditions:  $P_0=1013 \text{ hPa}$ ,  $T_0=15^\circ\text{C}$

Troposphere (0-11 km):  $T=T_0+\Gamma z$  with  $\Gamma=-6.5^\circ\text{C}/\text{km}$

$$T(z) = T_0 + \Gamma z \quad dp/p = \frac{-gM_{air}}{R^*T(z)} dz$$

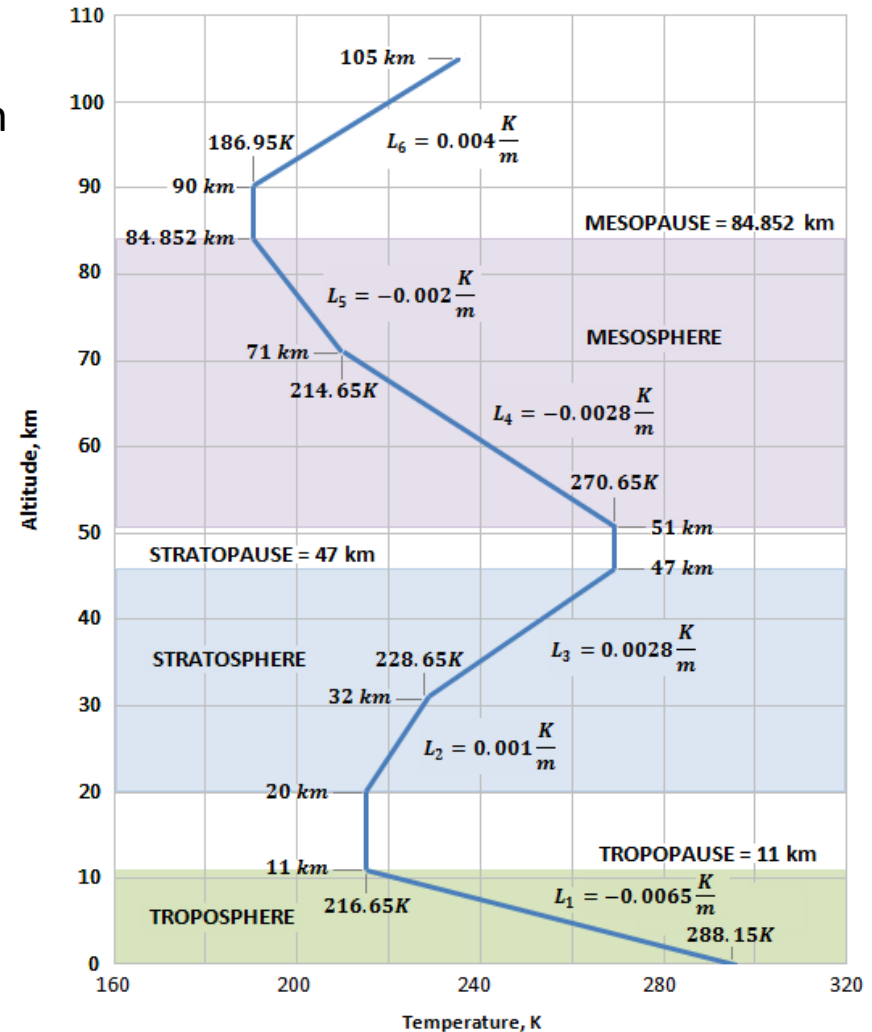
$$d(T_0 + \Gamma z) = \Gamma dz$$

$$dp/p = \frac{-gM_{air}}{\Gamma R^*} \frac{d(T_0 + \Gamma z)}{(T_0 + \Gamma z)}$$

$$\frac{P}{P_0} = \left( \frac{T_0 + \Gamma z}{T_0} \right)^{-\frac{M_a g}{\Gamma R}}$$

$\Gamma \Leftrightarrow$  Environmental lapse rate

Definition of standard temperatures at different altitudes



## Dry adiabatic lapse rate

First law of thermodynamics:  $dU = \delta Q + \delta W$

$$\delta Q = TdS = C_v dT + pdV$$

For adiabatic transformation:  $\delta Q = 0 = C_v dT + pdV$

$$pV = nR^*T \rightarrow pdV + Vdp = nR^*dT$$

$$0 = (nR^* + C_v)dT - Vdp$$

$$C_p dT = Vdp$$

$$dp = -\rho_{air}gdz = -\frac{m_{air}}{V}gdz$$

$$V = \frac{m_{air}RT}{p}$$

$$c_p = \frac{C_p}{m_{air}}$$

$$\frac{dT}{dz} = -\frac{g}{c_p}$$

9.81 m/s<sup>2</sup>

1006 J/(kg.K)

$$dp/p = c_p/R \times dT/T$$

$$\int_{standard}^{T,p}$$

$$\Gamma' = \frac{dT}{dz} \approx -10K/km$$

Adiabatic laspe rate

$$\theta = T_0 = T \left( \frac{p_{standard}}{p} \right)^{R/c_p}$$

Potential temperature ↔ Temperature that the air parcel would acquire if adiabatically brought to a standard pressure

## Vertical stability

Vertical momentum equation for the displaced air parcel:

$$(z) \quad \rho_p \frac{d^2 \delta z_p}{dt^2} = -\rho_p g - \frac{\partial p_a}{\partial z}$$

... combined with hydrostatic equation applied to ambient air:

$$\rho_a g + \frac{\partial p_a}{\partial z} = 0$$

Gas law &  $p_a = p_p$

$$\Rightarrow a_p = \frac{d^2 \delta z_p}{dt^2} = g \frac{\rho_a - \rho_p}{\rho_p} = g \frac{T_p - T_a}{T_a}$$

With

$$\left. \begin{aligned} T_p &= T_s + \delta T_p \\ T_a &= T_s + \delta T_a \end{aligned} \right\}$$

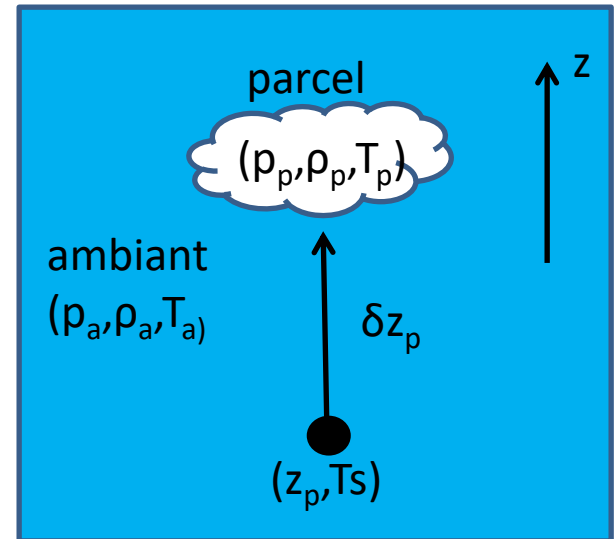
$$a_p = g \frac{\delta T_p - \delta T_a}{T_a}$$

$$\begin{aligned} \theta &= T \left( \frac{p_0}{p} \right)^\kappa \\ \frac{\delta \theta}{\theta} &= \frac{\delta T}{T} - \kappa \frac{\delta p}{p} \\ \delta \theta_p &= 0 \end{aligned}$$

$$a_p = \frac{g}{T_a} \left[ \left( \frac{\partial T}{\partial z} \right)_p - \left( \frac{\partial T}{\partial z} \right)_a \right] \delta z_p$$

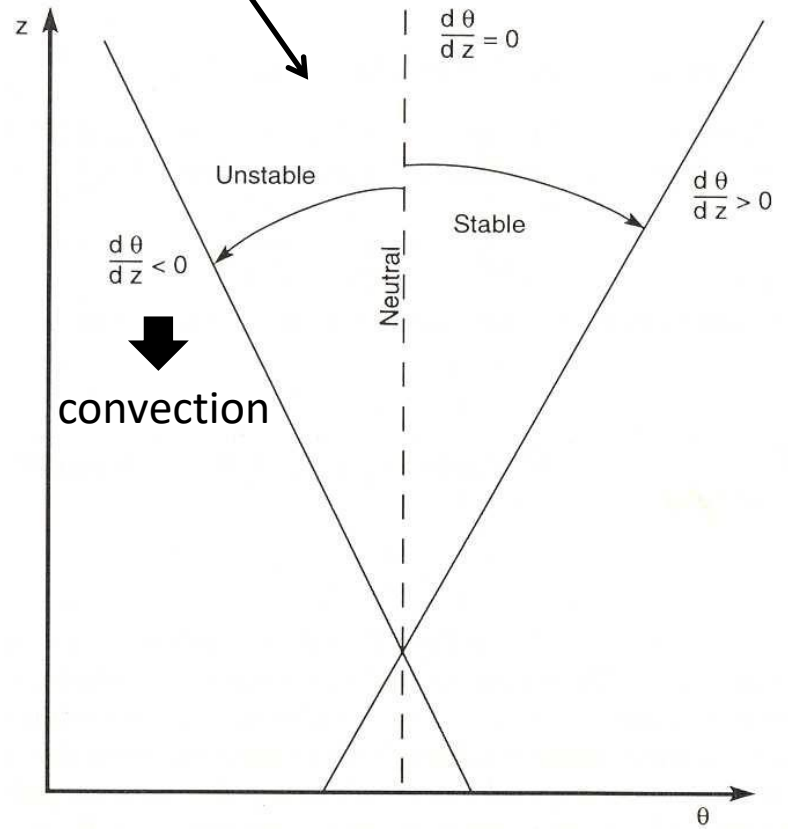
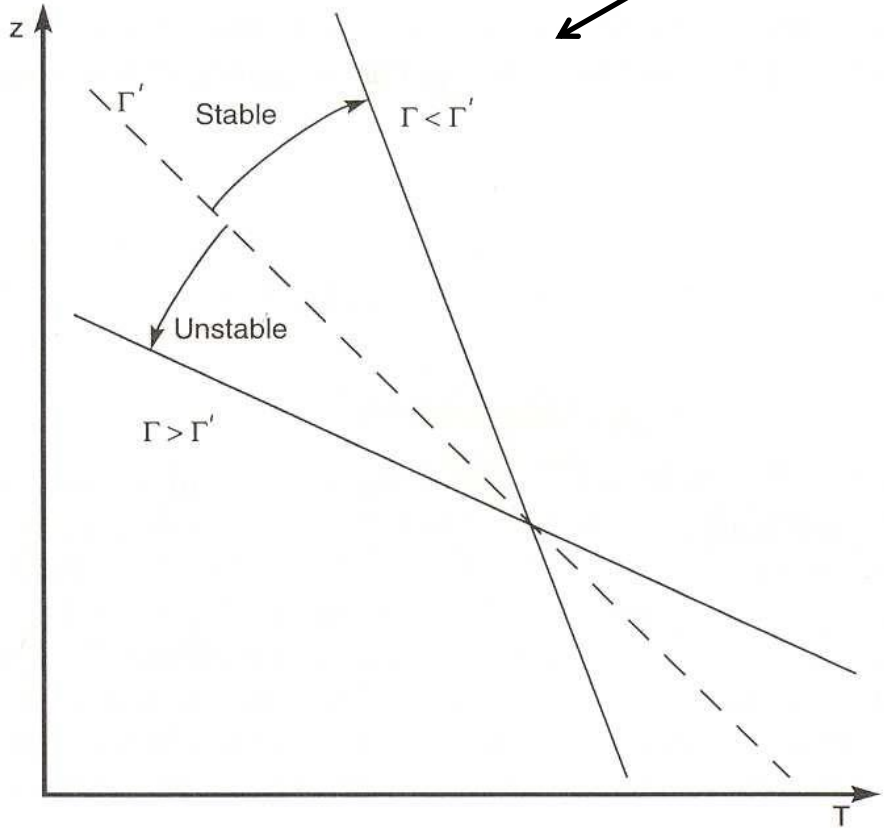
$\Gamma'$  (adiabatic)     $\Gamma$

$$a_p = -g \frac{\delta \theta_a}{\theta_a} = -\frac{g}{\theta_a} \left( \frac{\partial \theta_a}{\partial z} \right) \delta z_p$$

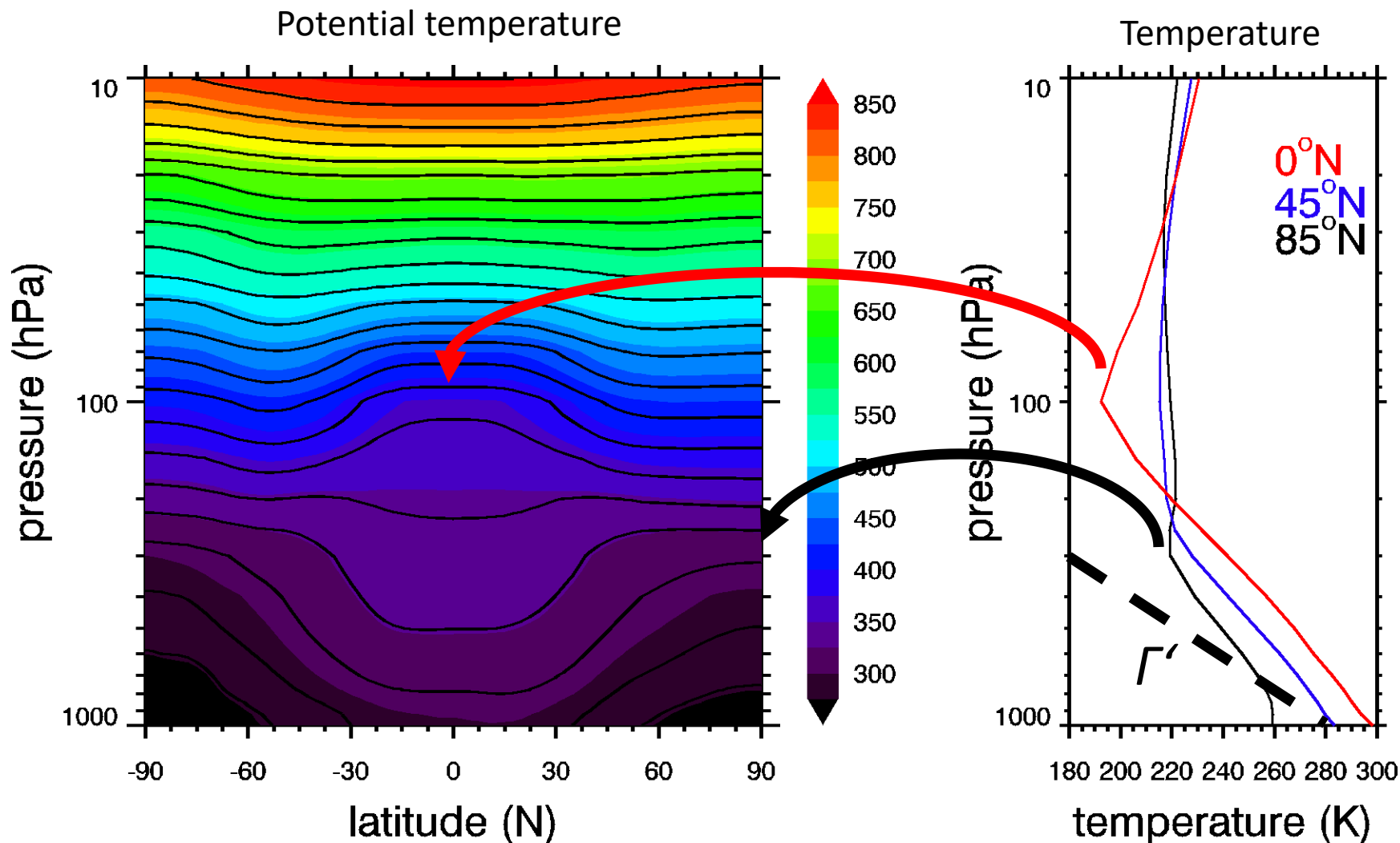


## Vertical stability: application

$$\frac{d^2 \delta z_p}{dt^2} = \frac{g}{T_a} [\Gamma' - \Gamma_a] \delta z_p = -\frac{g}{\theta_a} \left( \frac{\partial \theta_a}{\partial z} \right) \delta z_p$$



# Potential temperature mean vertical profile



## Atmospheric boundary layer diurnal cycle

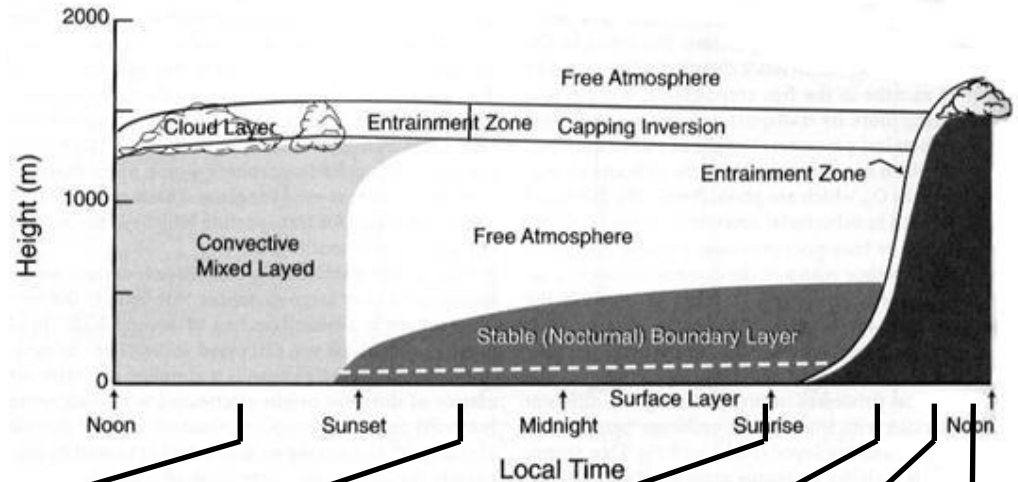
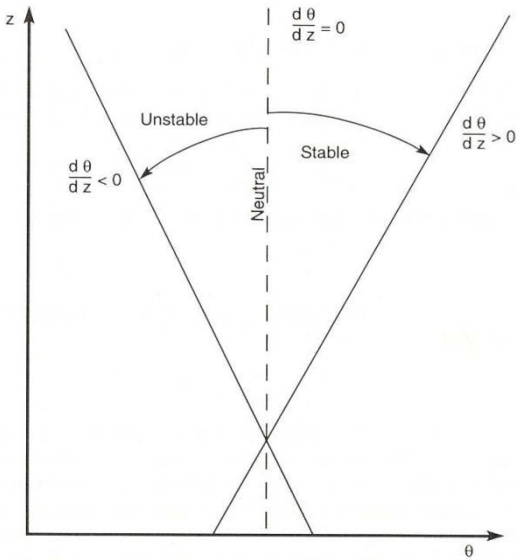
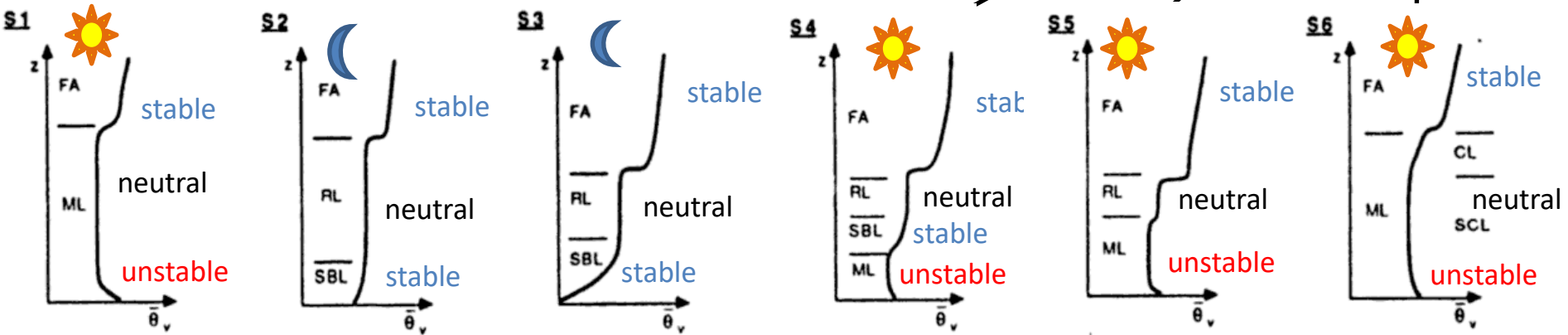


Figure 2.7. Typical diurnal evolution of the planetary boundary layer under a high pressure system. (adapted from Stull, Boundary Layer Meteorology, Kluwer, 1988)



## Brünt-Vaissala frequency : acceleration along z axis

$$\underbrace{\frac{d^2 \delta z_p}{dt^2} = -\frac{g}{\theta_a} \left( \frac{\partial \theta_a}{\partial z} \right) \delta z_p}_{\text{Harmonic oscillator !}} = -N^2 \delta z_p, \text{ with}$$

$$N^2 = \frac{g}{\theta_a} \left( \frac{\partial \theta_a}{\partial z} \right)$$

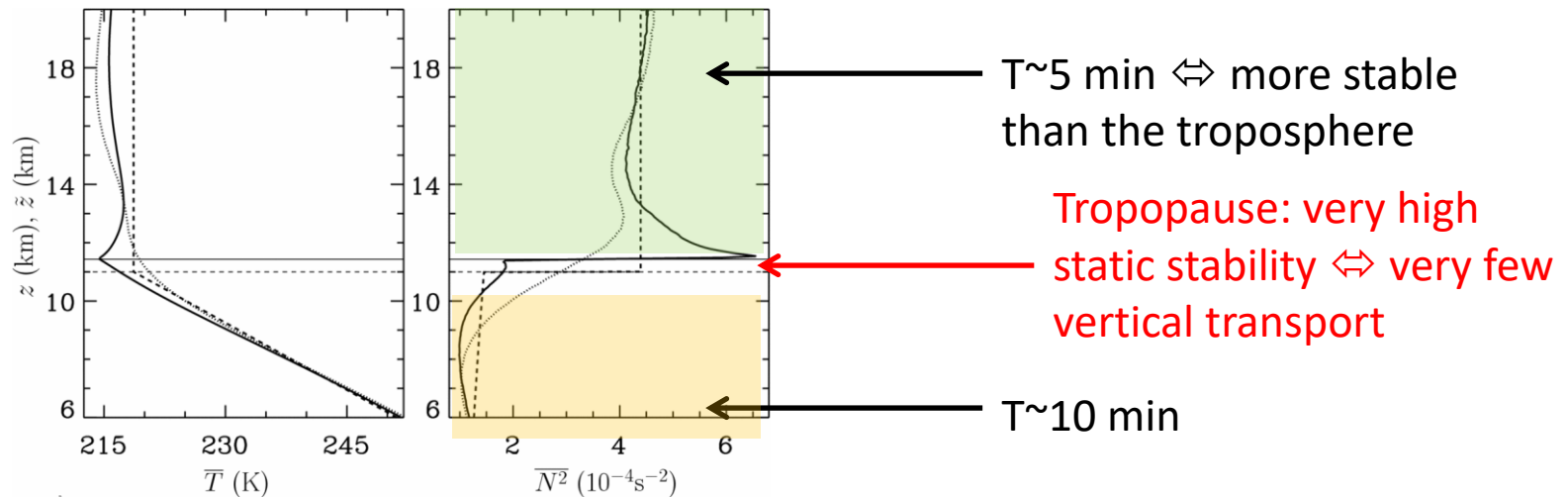
N is the Brünt-Vaissala frequency

Harmonic oscillator !

if  $\frac{\partial \theta_a}{\partial z} < 0 \Leftrightarrow$  unstable: Convection

if  $\frac{\partial \theta_a}{\partial z} > 0$  then  $N \in \mathbb{R} \Leftrightarrow$  stable: the parcel oscillates with the frequency N or the period  $T=2\pi/N$

Radiosounding at midlatitudes from 1998 to 2002



(Birner, 2006)

## Atmospheric water vapor

- Water vapour mass mixing ratio:

$$r = \frac{m_v}{m_{dry\ air}} = \frac{M_v}{M_{dry\ air}} \frac{P_v}{P_{dry\ air}}$$

Water vapour partial pressure

$$r = 0.622 \frac{P_v}{P - P_v}$$

with  $P = P_{dry\ air} + P_v$

- Saturated air:

$$\frac{1}{P_{v,sat}} \frac{dP_{v,sat}}{dT} = \frac{L(T)}{R_v T^2}$$

with  $R_v = R^*/M_v$   
 $L$  : specific latent heat of vaporization

Clausius-Clapeyron



$$P_{v,sat} = 6.112 e^{\left(17.67 \frac{T-273.15}{T-29.65}\right)}$$

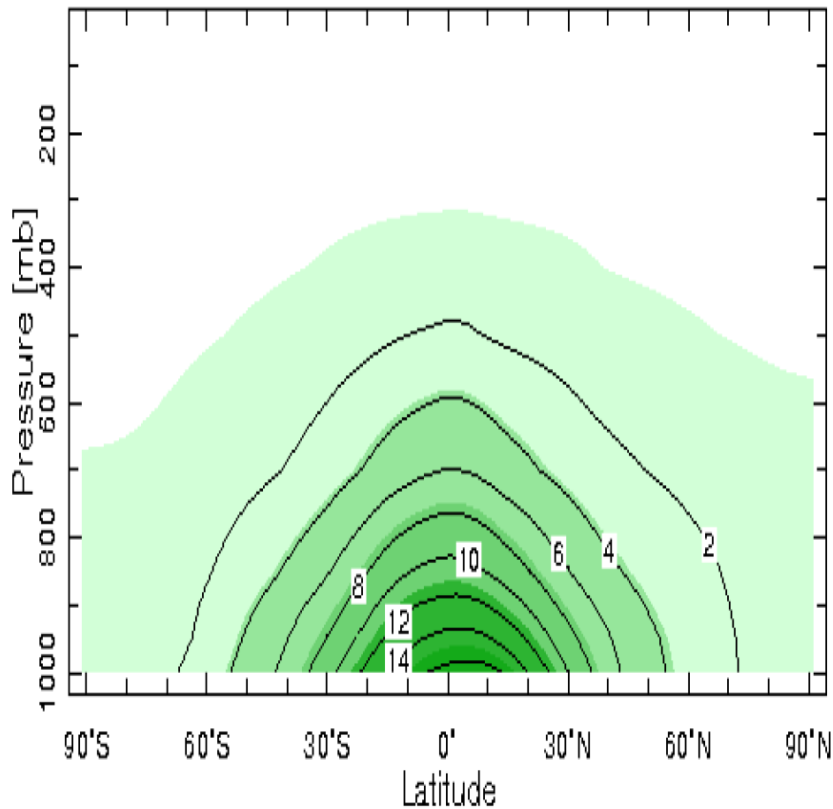
- @1000 hPa, 20°C :  $r_{sat} = 14.5$  g/kg
- @500 hPa, -30°C :  $r_{sat} = 0.47$  g/kg
- @100 hPa, -80°C :  $r_{sat} = 0.003$  g/kg

Clausius-Clapeyron law limits the air water vapor content by fixing the saturated pressure as a function of T and P.



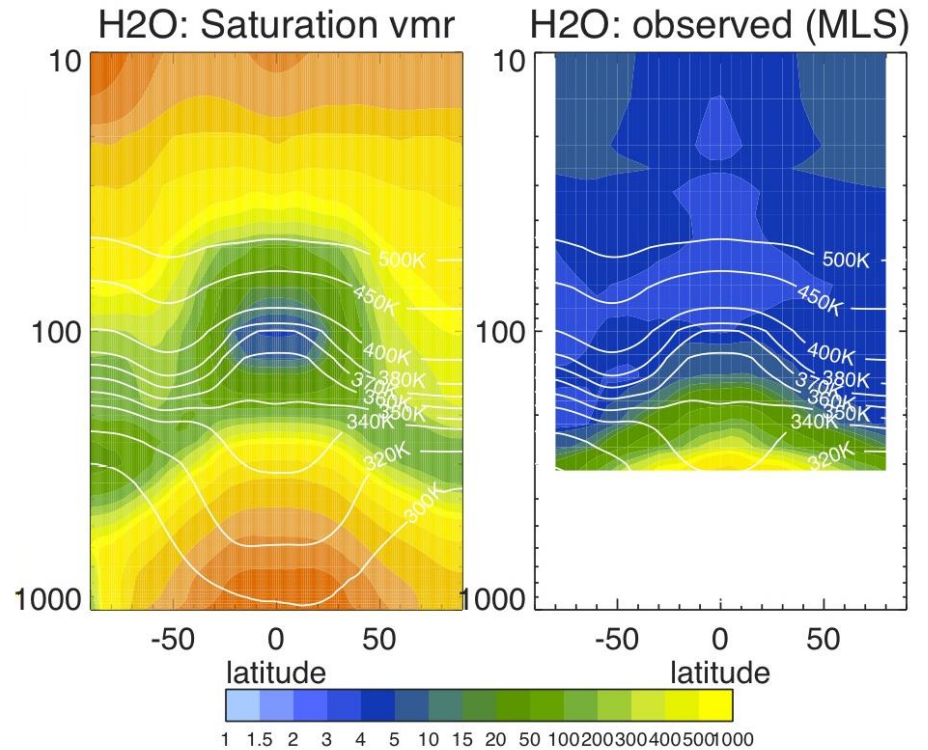
# Atmospheric water vapor (2)

Mean water vapor mass mixing ratio distribution (g/kg) (troposphere)



Courtesy F. Codron

Water vapor volume mixing (stratosphere)



! The stratosphere is extremely dry !

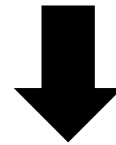
## Saturated lapse rate

Dry adiabatic conditions:  $\delta Q = 0 = C_p dT + g dz$

If the water vapor in the air parcel is condensing during the ascent:  $\delta Q = -L dr$

### Pseudo-adiabatic hypothesis:

The water vapour (liquid or vapour) is neglected but the latent heat release is considered.



(r decreases if condensation:  
r mass vapor mixing ration)

$$0 = C_p dT + g dz + L dr$$

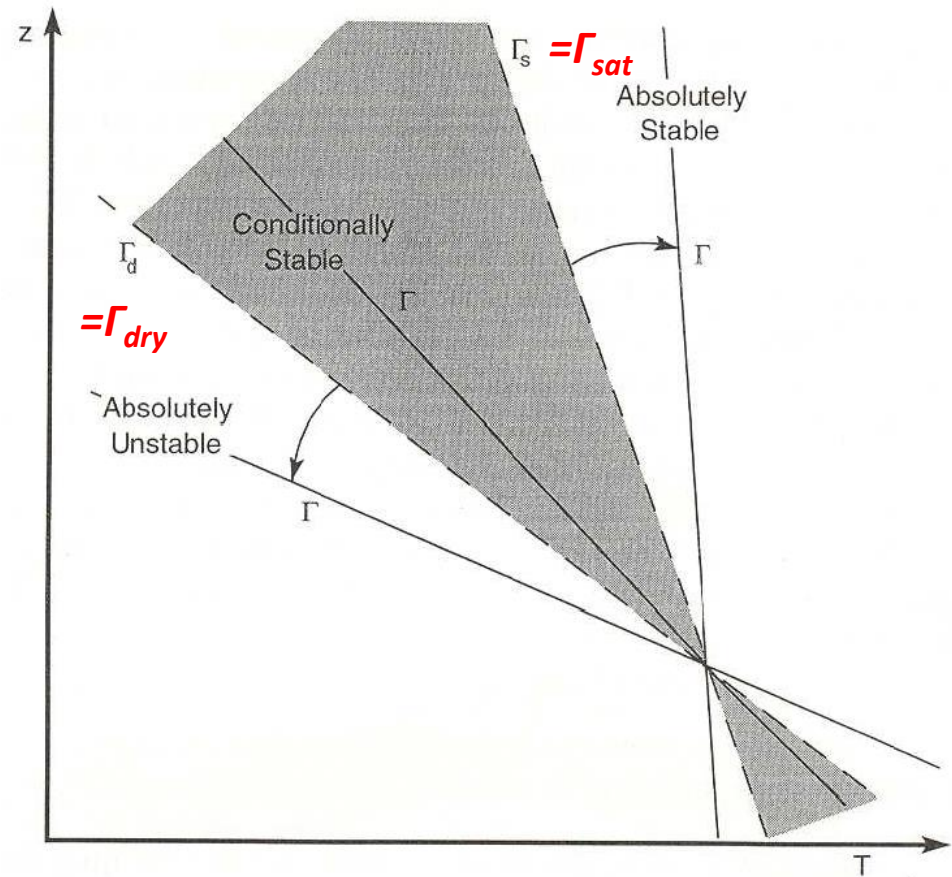
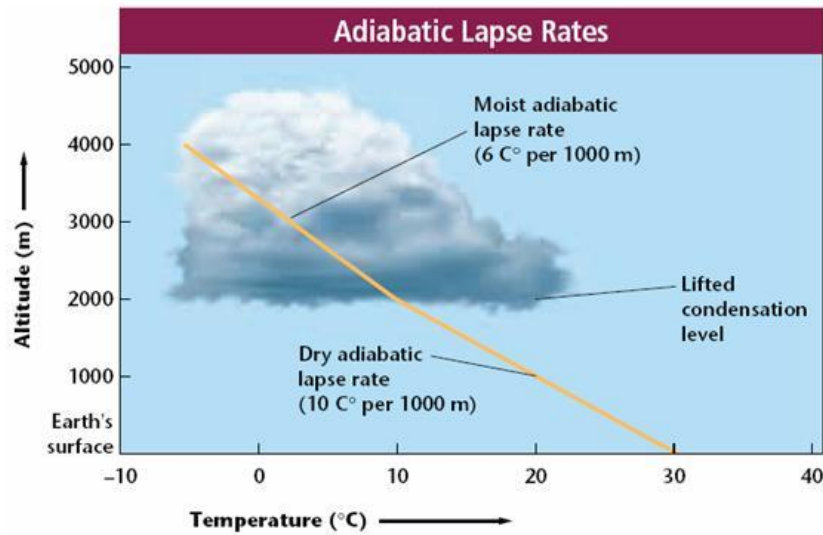
for saturated air:  $r = r_{sat}(T, p)$  , thus  $dr_{sat}(T, p) = \frac{\partial r_{sat}}{\partial T} dT + \frac{\partial r_{sat}}{\partial p} dp$

$$\Rightarrow \left( C_p + L \frac{\partial r_{sat}}{\partial T} \right) = -g \left( 1 - \rho L \frac{\partial r_{sat}}{\partial p} \right) \frac{dz}{dT}$$

$$\Rightarrow \Gamma_{sat} = \frac{dT}{dz_{sat}} = \Gamma_{dry} \frac{\left( 1 - \rho L \frac{\partial r_{sat}}{\partial p} \right)}{\left( 1 + \frac{L}{C_p} \frac{\partial r_{sat}}{\partial T} \right)}$$

$\Gamma_{sat} > \Gamma_{dry}$

## Conditional stability



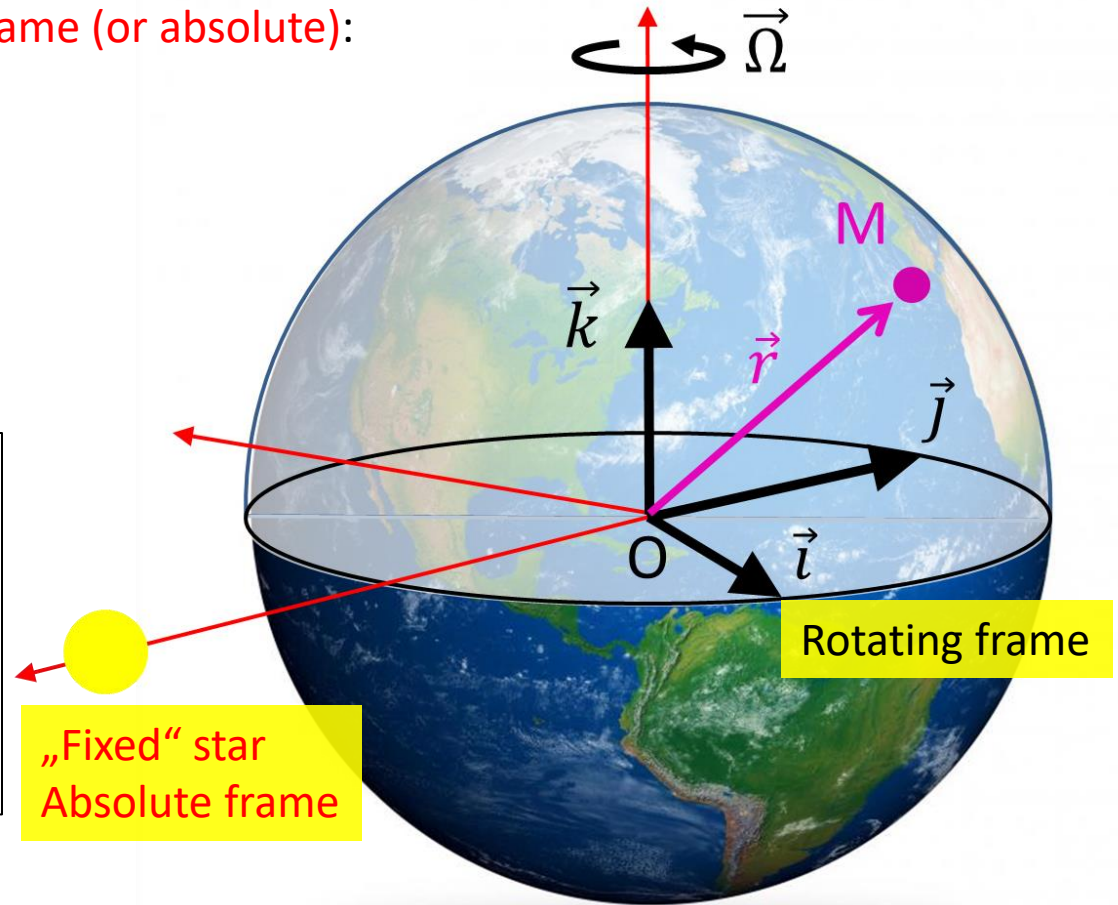
# III. Atmospheric Circulation

## Frames of reference

Newton's Second Law in a **inertial frame (or absolute)**:

$$\vec{a}_a = \sum \frac{\vec{F}}{m}$$

We want to express this in a reference frame which rotates with the Earth



## Absolute acceleration

$$\vec{a}_a = \frac{D\vec{V}_a}{Dt} = \frac{D^2\vec{r}}{Dt^2} \quad \text{with} \quad \vec{r} = \overline{OM} = x\vec{i} + y\vec{j} + z\vec{k}$$

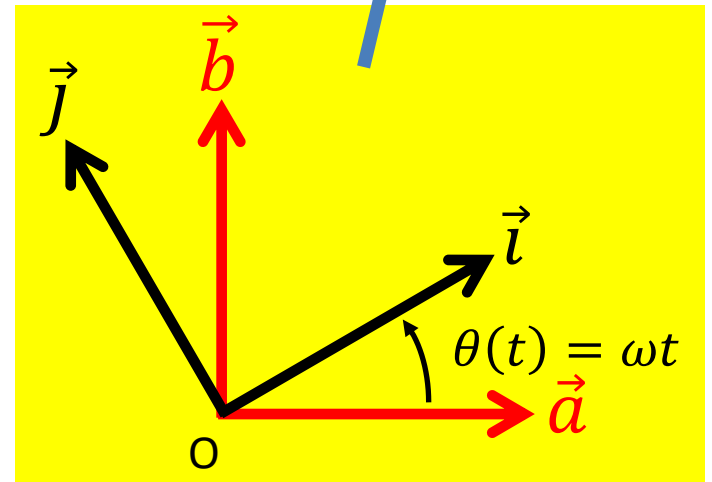
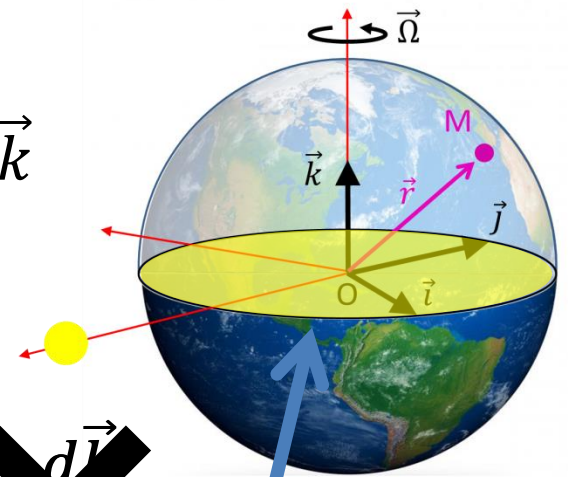
(D/Dt to distinguish the total derivative in the absolute frame)

$$\vec{V}_a = \frac{D\vec{r}}{Dt} = \frac{dx}{dt}\vec{i} + x\frac{d\vec{i}}{dt} + \frac{dy}{dt}\vec{j} + y\frac{d\vec{j}}{dt} + \frac{dz}{dt}\vec{k} + z\frac{d\vec{k}}{dt}$$

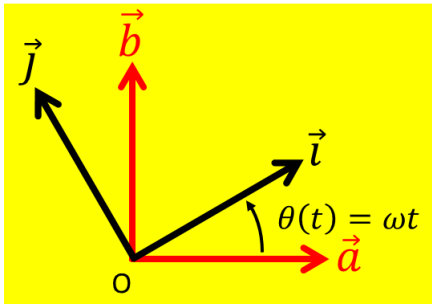
$$\vec{V}_a = \vec{V}_r + x\frac{d\vec{i}}{dt} + y\frac{d\vec{j}}{dt} \quad ?$$

Absolute velocity

Relative velocity



## Absolute acceleration (2)



$$\rightarrow \begin{cases} \vec{i} = \cos \theta \vec{a} + \sin \theta \vec{b} \\ \vec{j} = -\sin \theta \vec{a} + \cos \theta \vec{b} \end{cases} \rightarrow \begin{cases} d\vec{i}/dt = \omega \vec{j} \\ d\vec{j}/dt = -\omega \vec{i} \end{cases}$$

We thus find

$$x \frac{d\vec{i}}{dt} + y \frac{d\vec{j}}{dt} = x\omega \vec{j} - y\omega \vec{i}$$

and  $\vec{\Omega} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}, \vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  thus  $\vec{\Omega} \times \vec{r} = x\omega \vec{j} - y\omega \vec{i}$

Finally:  $\vec{V}_a = \vec{V}_r + x \frac{d\vec{i}}{dt} + y \frac{d\vec{j}}{dt} = \vec{V}_r + \vec{\Omega} \times \vec{r}$

Absolute velocity = Relative velocity + Entrainment velocity

## Absolute acceleration (3)

$$\vec{a}_a = \frac{D\vec{V}_a}{Dt} = \frac{D\vec{V}_r}{Dt} + \frac{D}{Dt}(\vec{\Omega} \times \vec{r}) = \frac{D\vec{V}_r}{Dt} + \frac{D\vec{\Omega}}{Dt} \times \vec{r} + \vec{\Omega} \times \frac{D\vec{r}}{Dt}$$

$$\vec{a}_a = \vec{a}_r + \vec{\Omega} \times \vec{V}_r + D\vec{\Omega}/Dt \times \vec{r} + \vec{\Omega} \times (\vec{V}_r + \vec{\Omega} \times \vec{r})$$

$$\vec{a}_a = \vec{a}_r + \underbrace{2\vec{\Omega} \times \vec{V}_r}_{\text{Coriolis acceleration}} + \underbrace{D\vec{\Omega}/Dt \times \vec{r}}_{\substack{\text{Euler} \\ \text{acceleration} \\ =0}} + \underbrace{\vec{\Omega} \times (\vec{\Omega} \times \vec{r})}_{\text{Centripetal acceleration}} = \sum \vec{F}/m$$

In practice  
( $V_r=V$ )

$$\frac{d\vec{V}}{dt} = -2\vec{\Omega} \times \vec{V} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + \sum \vec{F}/m$$

Coriolis  
„apparent“ force

Centrifugal  
„apparent“ force

Fundamental forces  
???



## Fundamental forces: Pressure gradient

$$\left\{ \begin{array}{l} p_A = p_0 + \frac{\partial p}{\partial x} \frac{\delta x}{2} \\ p_B = p_0 - \frac{\partial p}{\partial x} \frac{\delta x}{2} \end{array} \right.$$

$$d\vec{F} = p \cdot d\vec{S}$$

$$\left\{ \begin{array}{l} F_{Ax} = - \left( p_0 + \frac{\partial p}{\partial x} \frac{\delta x}{2} \right) \delta y \delta z \\ F_{Bx} = + \left( p_0 - \frac{\partial p}{\partial x} \frac{\delta x}{2} \right) \delta y \delta z \end{array} \right.$$

$\delta V = \delta x \delta y \delta z$

$(x_0, y_0, z_0)$   
 $P_0$

$F_{Bx}$  → B      A ←  $F_{Ax}$

$\delta z$        $\delta y$        $\delta x$

$$m = \rho \delta V = \rho \delta x \delta y \delta z$$

$$F_x = F_{Ax} + F_{Bx} = - \frac{\partial p}{\partial x} \delta x \delta y \delta z \quad \longrightarrow \quad \frac{F_x}{m} = - \frac{1}{\rho} \frac{\partial p}{\partial x} \quad \longrightarrow \quad \frac{\vec{F}}{m} = - \frac{1}{\rho} \vec{\nabla} p$$

## Fundamental forces (2)

### ➤ Gravitational force & gravity

$$\vec{F}_g = -\frac{GMm}{r^2} \left( \frac{\vec{r}}{|\vec{r}|} \right)$$

$$\frac{\vec{F}_g}{m} \equiv \vec{g}^* = -\frac{GM}{r^2} \left( \frac{\vec{r}}{|\vec{r}|} \right)$$

$$\vec{g}^* = -\frac{GM}{(a+z)^2} \left( \frac{\vec{r}}{|\vec{r}|} \right) = \frac{\vec{g}_0^*}{(1+z/a)^2} \quad , \text{with} \quad \vec{g}_0^* = -\frac{GM}{(a)^2} \left( \frac{\vec{r}}{|\vec{r}|} \right)$$

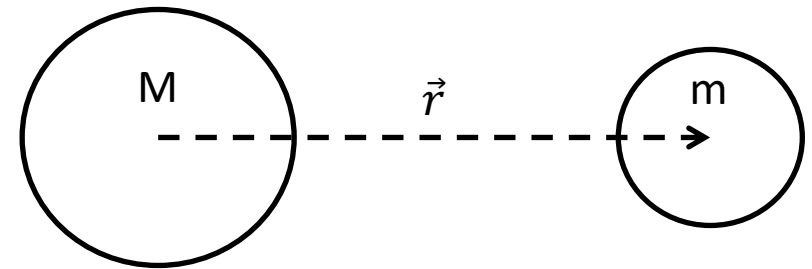
In practice:  $\vec{g}_0^* = \vec{g}^*$

Gravity = Gravitational + Centrifugal forces

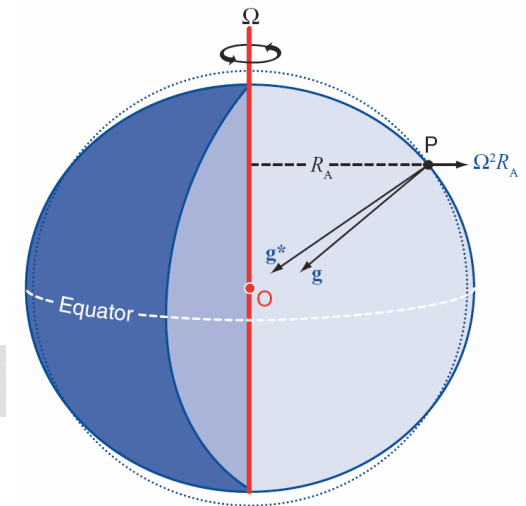
$$\vec{g} \equiv \vec{g}^* + \Omega^2 \vec{R}_A \approx 9.81 \text{ m/s}^2$$

### ➤ Friction

$$\vec{F}_f = -\alpha \vec{V} \longleftarrow \text{Wind vector}$$



a: Earth's radius  
z: altitude  
G: gravitational constant

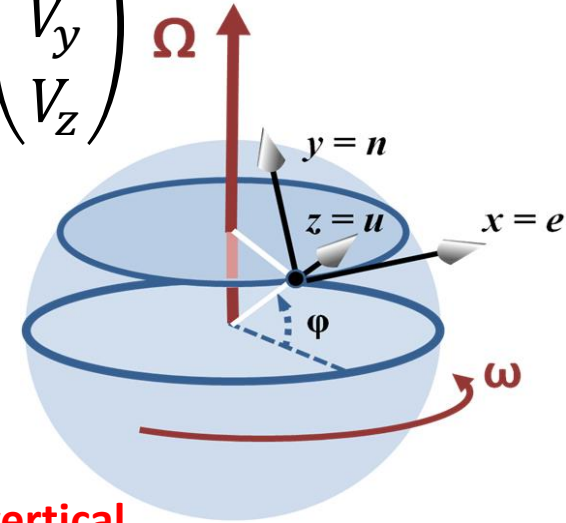


$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

## Coriolis effect

$$\vec{a}_c = -2\vec{\Omega} \times \vec{V} \quad \text{with} \quad \vec{\Omega} = \omega \begin{pmatrix} 0 \\ \cos \varphi \\ \sin \varphi \end{pmatrix}, \vec{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\vec{a}_c = 2\omega \begin{pmatrix} V_y \sin \varphi - \cancel{V_z \cos \varphi} \\ -V_x \sin \varphi \\ \cancel{V_x \cos \varphi} \end{pmatrix}$$



In atmospheric dynamics, the **vertical velocity is small** and the **vertical component of the Coriolis acceleration is small compared to gravity**

$$\vec{a}_c = 2\omega \sin \varphi \begin{pmatrix} V_y \\ -V_x \end{pmatrix} = f \begin{pmatrix} V_y \\ -V_x \end{pmatrix} = -f\vec{k} \times \vec{V}_h$$

Ballistic missile fired eastward at 43°N ( $f=10^{-4} \text{ s}^{-1}$ ). If the missile travels 1000 km at  $V_{x0}=1000 \text{ m/s}$ , how much is the missile deflected ?

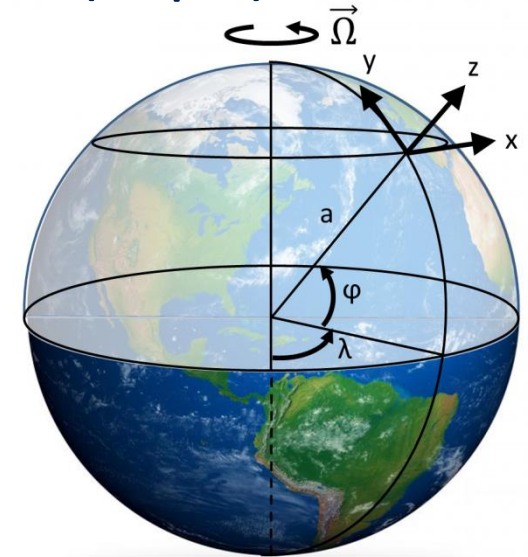
$$\frac{dV_y}{dt} = -fV_{x0} \int \rightarrow V_y = -fV_{x0}t \int \rightarrow \delta y = -\frac{fV_{x0}t^2}{2} = -50 \text{ km (southward)}$$

## Momentum equation: Spherical coordinates ( $\lambda, \varphi, z$ )

$$\frac{d\vec{V}}{dt} = \vec{g} - 2\vec{\Omega} \times \vec{V} - \frac{1}{\rho} \vec{\nabla} p - \vec{F}_f$$

We note:  $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

In spherical coordinate, some **additional curvature terms** appear



$$\left\{ \begin{array}{l} \frac{du}{dt} - \frac{uv \tan \varphi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \varphi - 2\Omega w \cos \varphi + F_{fx} \quad (x) \\ \frac{dv}{dt} - \frac{u^2 \tan \varphi}{a} + \frac{vw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega u \sin \varphi + F_{fy} \quad (y) \\ \frac{dw}{dt} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + 2\Omega u \cos \varphi + F_{fz} \quad (z) \end{array} \right.$$

**! Very difficult to handle since nonlinear...**

# Momentum equation: horizontal scale analysis

... however, not all terms are important, it depends on the scale that is considered !

For synoptic scale at mid-latitudes ( $\varphi=45^\circ$ ):

- $U \sim 10 \text{ m/s}$ ,  $W \sim 1 \text{ cm/s}$ ,  $L \sim 10^6 \text{ m}$ ,  $D \sim 10^4 \text{ m}$ ,  $\Delta_H P / \rho \sim 10^3 \text{ m}^2/\text{s}^2$ ,  $T = L/U \sim 10^5 \text{ s}$ ,  $a \sim 6400 \text{ km}$ .
- $2\Omega \sin(\varphi) \sim 2\Omega \cos(\varphi) \sim 10^{-4} \text{ s}^{-1}$

Thus, considering the x&y-equation (and neglecting friction):

$$(x) \quad \frac{du}{dt} - \frac{uv \tan \varphi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \varphi - 2\Omega w \cos \varphi$$

$$(y) \quad \frac{dv}{dt} - \frac{u^2 \tan \varphi}{a} + \frac{vw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega u \sin \varphi$$

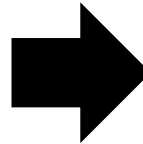
(scale)	$\frac{U^2}{L}$	$\frac{U^2}{a}$	$\frac{UW}{a}$	$\frac{\Delta p}{\rho L}$	$f_0 U$	$f_0 W$
(m/s <sup>2</sup> )	$10^{-4}$	$10^{-5}$	$10^{-8}$	$10^{-3}$	$10^{-3}$	$10^{-6}$

Coriolis force and pressure gradient force in approximate balance  $\Leftrightarrow$  geostrophic approximation

## Geostrophic balance

(x)-geostrophic  $\frac{1}{\rho} \frac{\partial p}{\partial x} = 2\Omega v_g \sin \varphi$

(y)-geostrophic  $\frac{1}{\rho} \frac{\partial p}{\partial y} = -2\Omega u_g \sin \varphi$



$$v_g = \frac{1}{\rho f} \frac{\partial p}{\partial x}$$

$$u_g = -\frac{1}{\rho f} \frac{\partial p}{\partial y}$$

$$f = 2\Omega \sin \varphi$$

Coriolis parameter

~ 80% of motion can be explained by the geostrophic approximation

General expression:  $-2\vec{\Omega} \times \vec{V}_g - \frac{1}{\rho} \nabla_h p = 0$



$$f \vec{k} \times \vec{V}_g = -\frac{1}{\rho} \nabla_h p$$



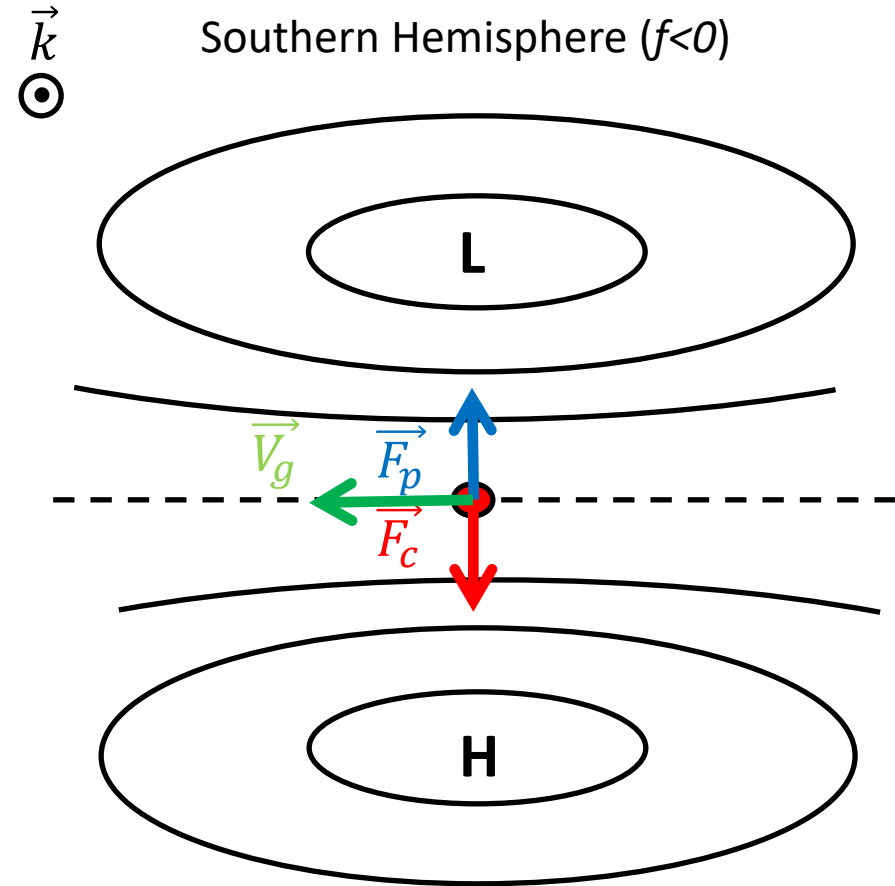
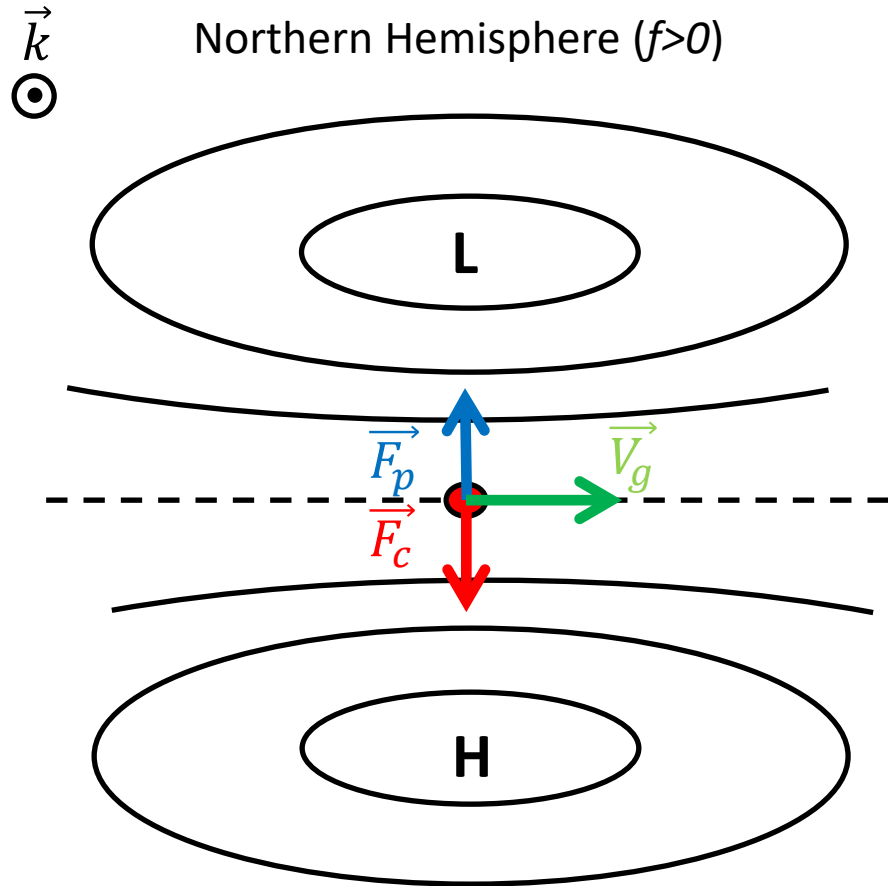
$$\times \vec{k} \text{ and } (\vec{A} \times \vec{B}) \times \vec{C} = (\vec{A} \cdot \vec{C}) \cdot \vec{B} - (\vec{B} \cdot \vec{C}) \cdot \vec{A}$$

$$\vec{V}_g = \frac{1}{\rho f} \vec{k} \times \nabla_h p$$

**The geostrophic wind blows along the isobares**

# Buys-Ballot law

$$\vec{V}_g = \frac{1}{\rho f} \vec{k} \times \nabla_h p$$



“In the Northern Hemisphere, if a person stands with his back to the wind, the atmospheric pressure is low to the left, high to the right”

## Validity of geostrophic approximate

To obtain prediction equation, acceleration term has to be retained

$$\left\{ \begin{array}{l} \frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv = f(v - v_g) \\ \frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu = -f(u - u_g) \end{array} \right.$$

Acceleration = Pressure gradient + Coriolis force

**Validity of the geostrophic approximation:** Acceleration/Coriolis  $\Leftrightarrow$  Rossby number

$R_0$  small

Significant Coriolis effect



$R_0$  big

No Coriolis effect



$$R_0 = \frac{U^2 / L}{f_0 U} = \frac{U}{f_0 L}$$

**The smallness of the Rossby number is a measure of the validity of the geostrophic approximation**



## Thermal wind

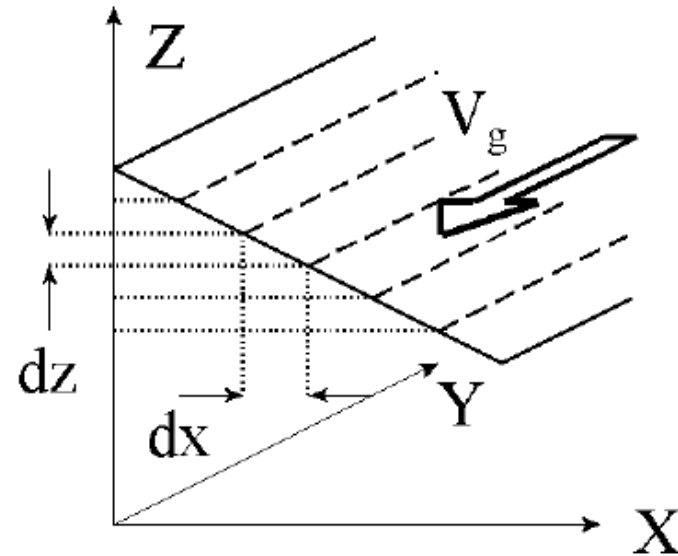
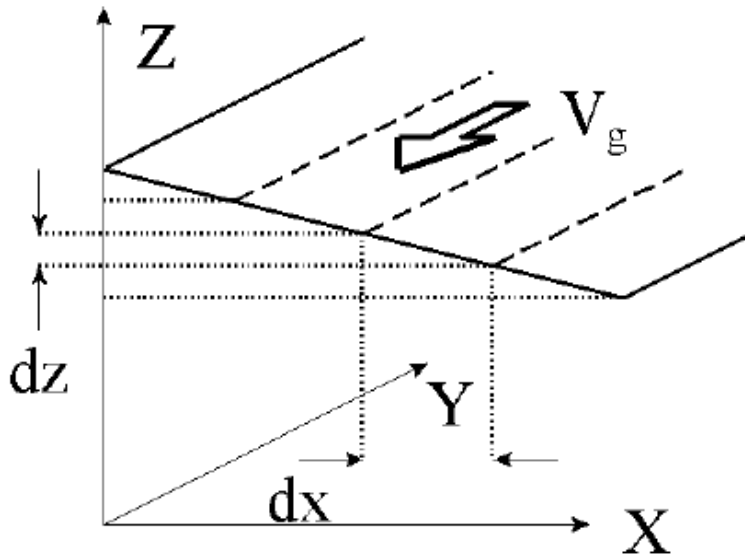
Starting from geostrophic wind:

$$u_g = -\frac{1}{\rho f} \left( \frac{\partial p}{\partial y} \right)_z \quad v_g = \frac{1}{\rho f} \left( \frac{\partial p}{\partial x} \right)_z$$

$$\downarrow \quad \frac{\partial p}{\partial z} = -\rho g$$

On a constant pressure surface:

$$u_g = -\frac{g}{f} \left( \frac{\partial z}{\partial y} \right)_p \quad v_g = \frac{g}{f} \left( \frac{\partial z}{\partial x} \right)_p$$



**The magnitude of the geostrophic wind is determined by the tilt of pressure surface**

## Thermal wind

By differentiating to pressure:

$$\frac{\partial}{\partial p} u_g = \frac{\partial}{\partial p} \left( -\frac{g}{f} \frac{\partial z}{\partial y} \right) = -\frac{g}{f} \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial p} \right)$$

$$\times p \quad \Downarrow \quad \frac{\partial z}{\partial p} = -\frac{1}{\rho g} = -\frac{1}{g} \frac{RT}{p}$$

$$p \frac{\partial}{\partial p} u_g = \frac{\partial u_g}{\partial \ln(p)} = -\frac{R}{f} \left( \frac{\partial T}{\partial y} \right)_p, \text{ and similarly } p \frac{\partial}{\partial p} v_g = \frac{\partial v_g}{\partial \ln(p)} = \frac{R}{f} \left( \frac{\partial T}{\partial x} \right)_p$$



Vectorial form:

$$\frac{\partial \vec{V}_g}{\partial \ln(p)} = -\frac{R}{f} \vec{k} \times \vec{\nabla}_p T$$

**Vertical variation of geostrophic wind depends on temperature horizontal gradient**

By integrating from  $p_0$  to  $p_1$  ( $p_0 > p_1$ ) we obtain the thermal wind equation:

$$u_T = -\frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial y} \right)_p \ln \left( \frac{p_0}{p_1} \right)$$

and

$$v_T = \frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial x} \right)_p \ln \left( \frac{p_0}{p_1} \right)$$

$\langle T \rangle$ : mean temperature in the  $[p_0, p_1]$  layer

**! The thermal wind equation is a relationship for the vertical wind shear !**

## Thermal wind: application to the jet stream

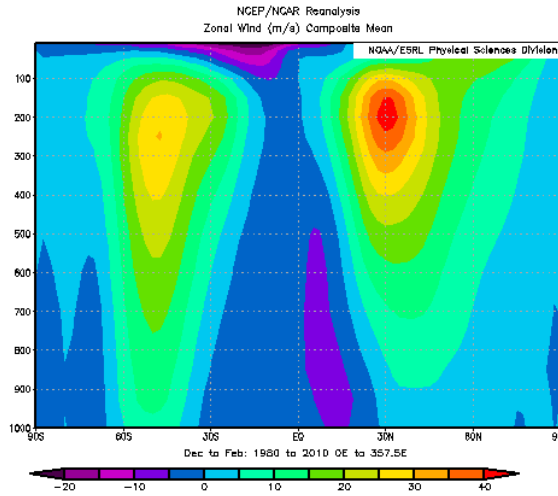
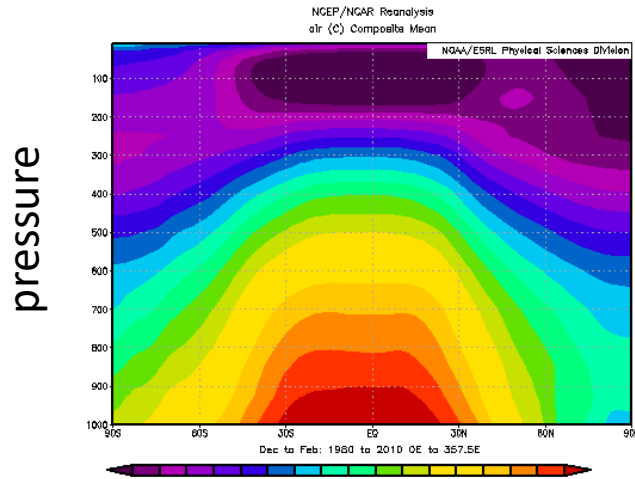
$$u_T = -\frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial y} \right)_p \ln \left( \frac{p_0}{p_1} \right)$$

NCEP reanalysis: 1980-2000

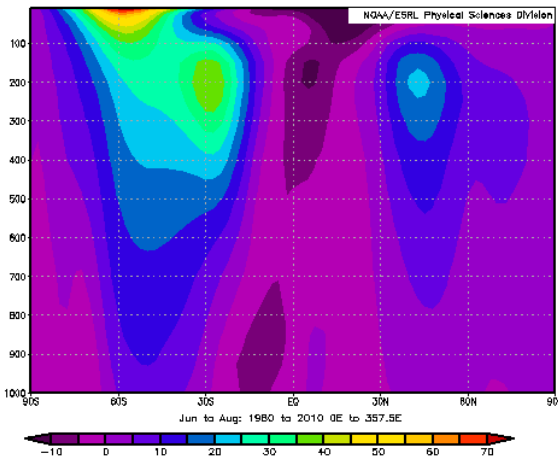
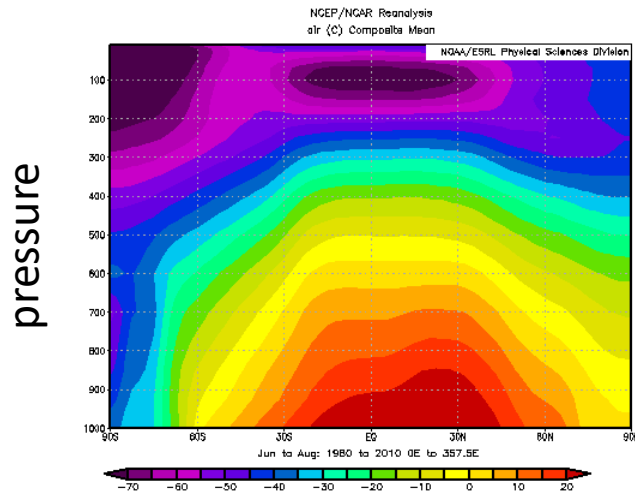
Temperature

Zonal wind

DJF



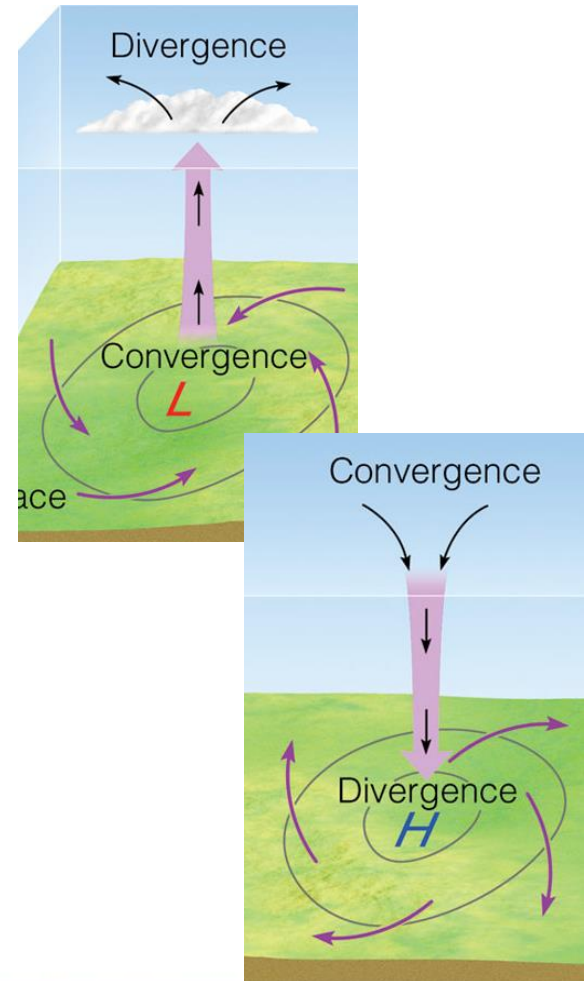
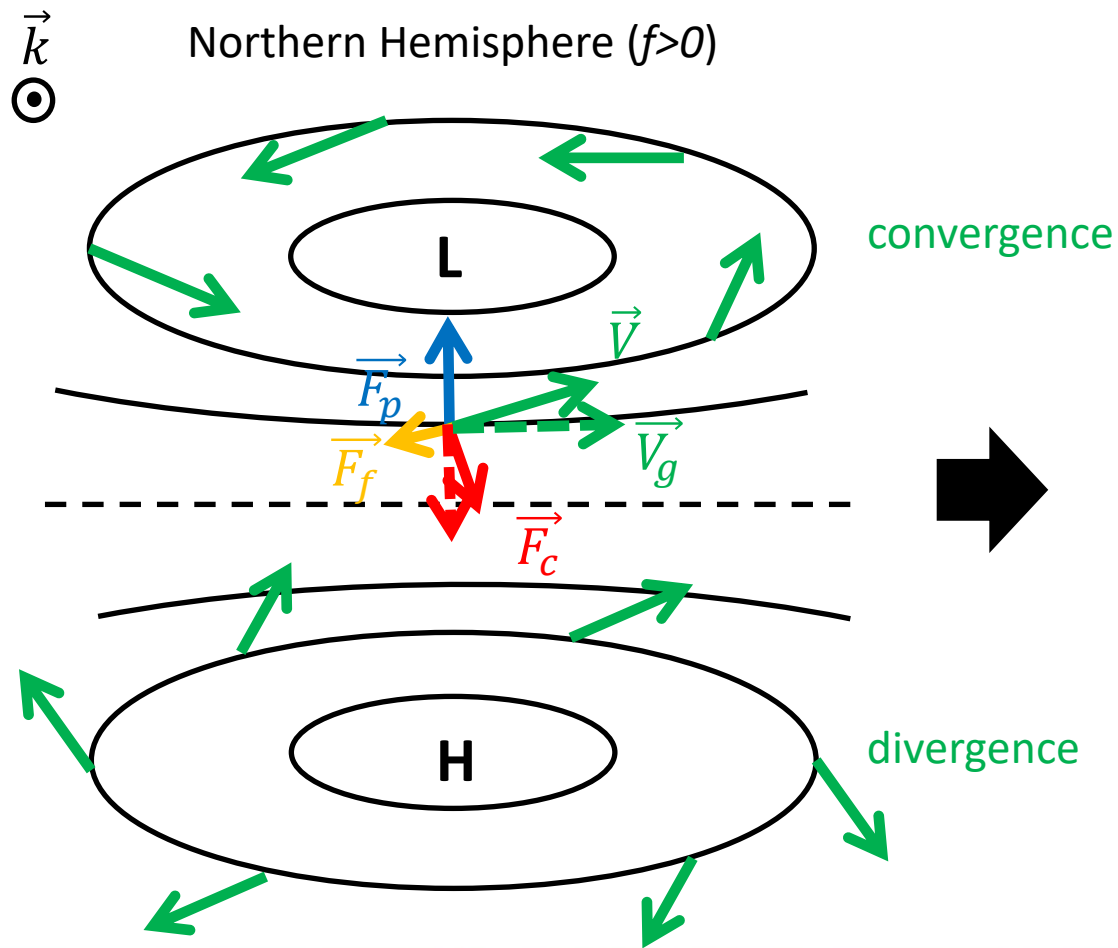
JJA



easterlies  
westerlies

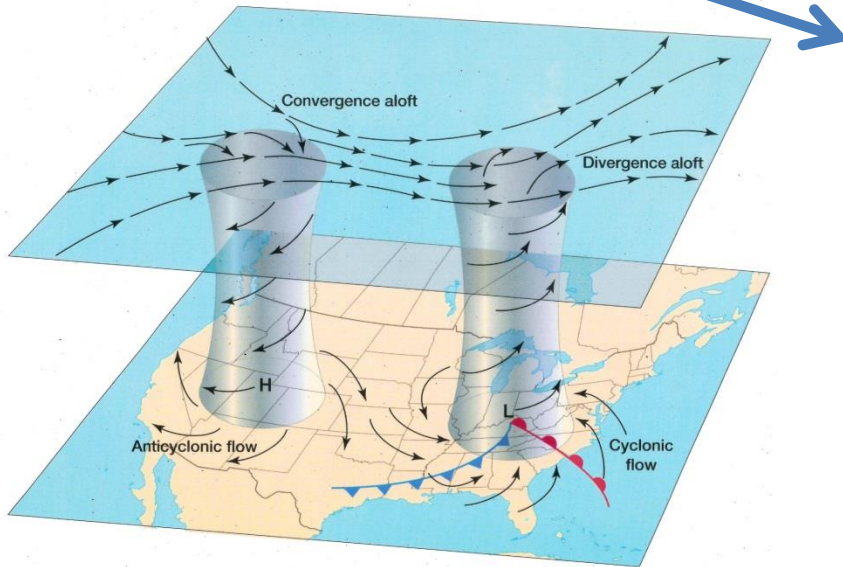
## Influence of friction: boundary layer

$$f\vec{k} \times \vec{V} + \vec{F}_f = -\frac{1}{\rho} \nabla_h p \quad , \text{with } \vec{F}_f = -\alpha \vec{V}$$

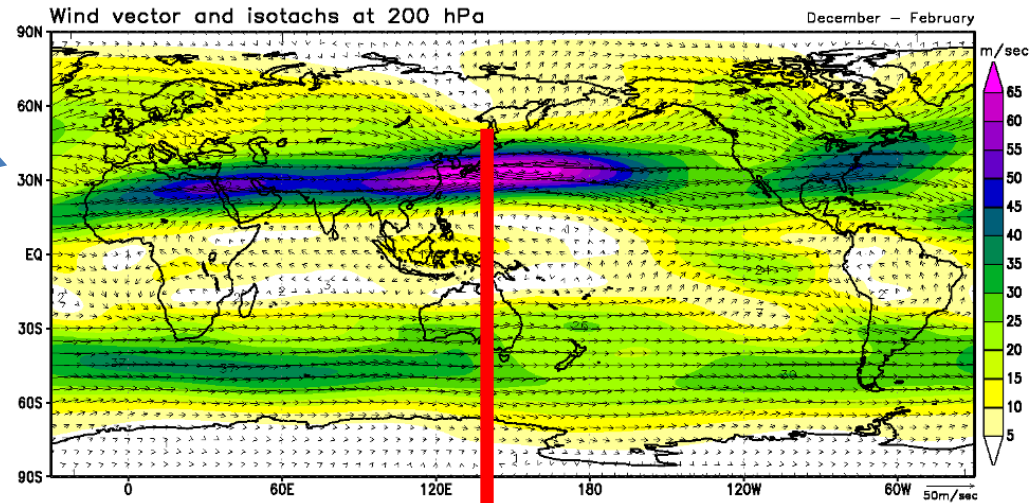


## A concrete example

Subtropical jetstream

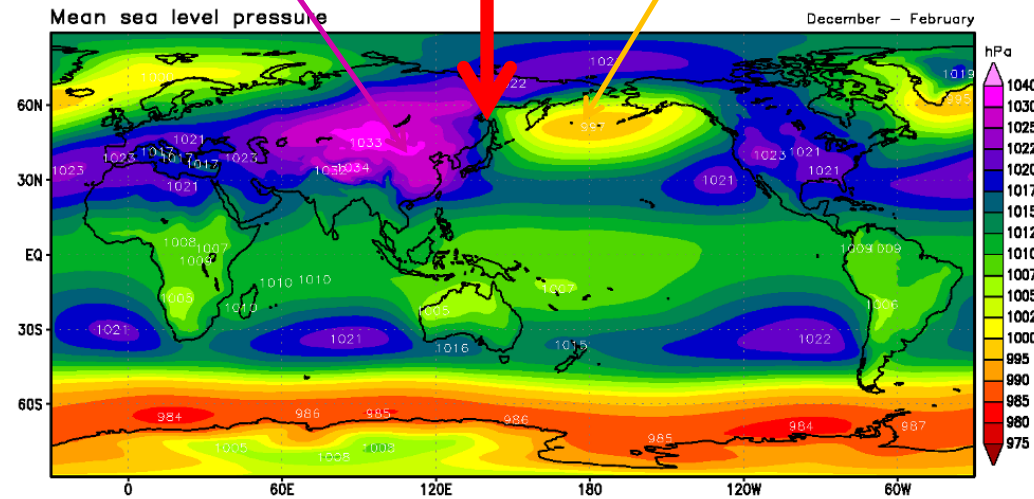


To characterize the atmospheric motions we can use also divergence and vorticity of the wind



Siberian high

Aleutian low



## The vorticity equation

Horizontal momentum equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (1)$$

$$\frac{\partial(2)}{\partial x} - \frac{\partial(1)}{\partial y} \quad \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (2)$$

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} + (\zeta + f) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \left( \frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) + v \frac{df}{dy}$$

$$= \frac{1}{\rho^2} \left( \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right)$$

$$\frac{df}{dt} = v \frac{df}{dy} \quad : f \text{ depends only on } y$$

with,  $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$$\frac{d(\zeta + f)}{dt} = -(\zeta + f) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left( \frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) - \frac{1}{\rho^2} \left( \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right)$$

$\zeta + f = \text{vorticity} + \text{planetary vorticity} = \text{absolute vorticity}$

## Vorticity equation : scale analysis

For synoptic scale motion at midlatitudes:

- $U \sim 10 \text{ m/s}$ ,  $W \sim 1 \text{ cm/s}$ ,  $L \sim 10^6 \text{ m}$ ,  $H \sim 10^4 \text{ m}$ ,  $\delta p \sim 10 \text{ hPa}$ ,  $\rho \sim 1 \text{ kg m}^{-3}$ ,  $\delta \rho / \rho \sim 10^{-2}$ .
- $f_0 = 10^{-4} \text{ s}^{-1}$

$$\frac{d(\zeta + f)}{dt} = -(\zeta + f) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \left( \frac{\partial w}{\partial x} \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) - \frac{1}{\rho^2} \left( \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} + \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right)$$

$\Downarrow$   
 $f_0 U / L = 10^{-9} \text{ s}^{-2}$

$\Downarrow$   
 $WU / (HL) = 10^{-11} \text{ s}^{-2}$

$\Downarrow$   
 $\delta \rho \delta \rho / (\rho^2 L^2) = 10^{-11} \text{ s}^{-2}$

$$\frac{d_h(\zeta + f)}{dt} = -(\zeta + f) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

### Link between the vorticity and the divergence of the flow

- same phenomenon as an ice skating turning on herself
- spreading her arms => decreases her rotating speed
  - tightening his arms => increases her rotating speed



## Potential vorticity

We can further show:

$$PV = (\zeta_{\theta} + f) \left( -g \frac{\partial \theta}{\partial p} \right) = const$$

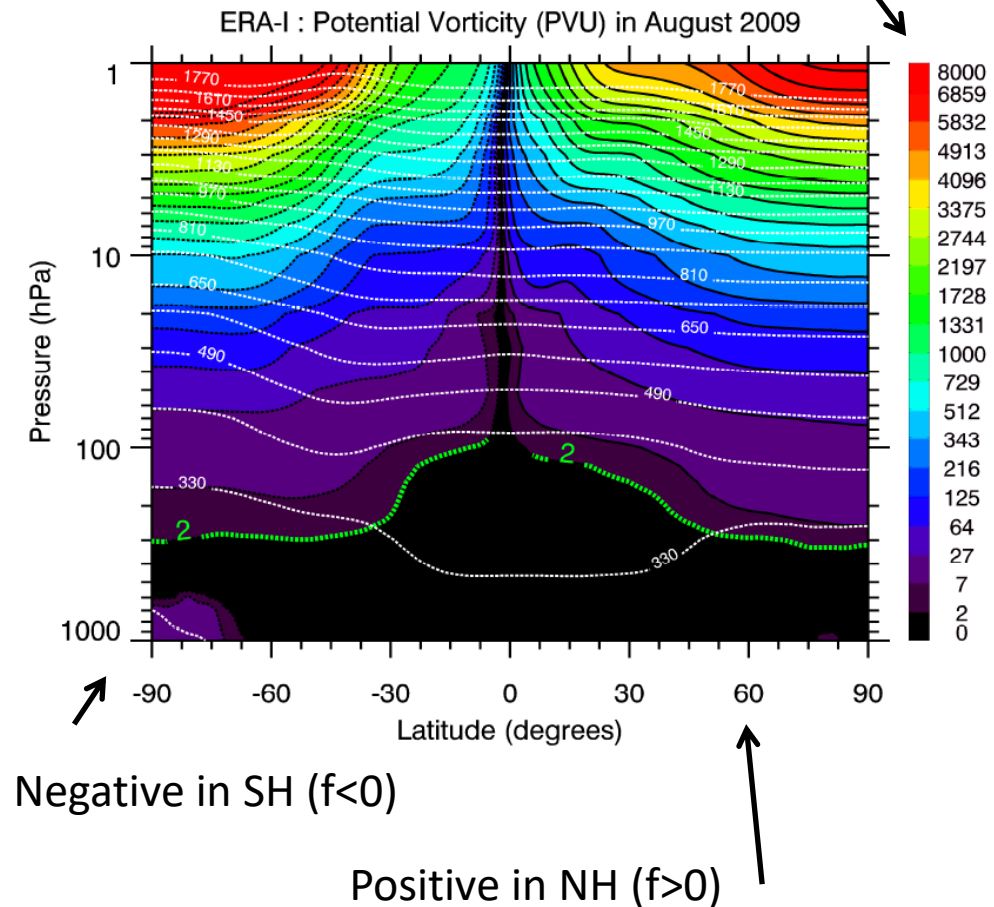
Absolute vorticity

Vertical stability

**The potential vorticity is conserved under adiabatic and frictionless motions**

- $f$  increases with latitude: meridional gradient
- $\theta$  increases with altitude: vertical gradient

Absolute PV values



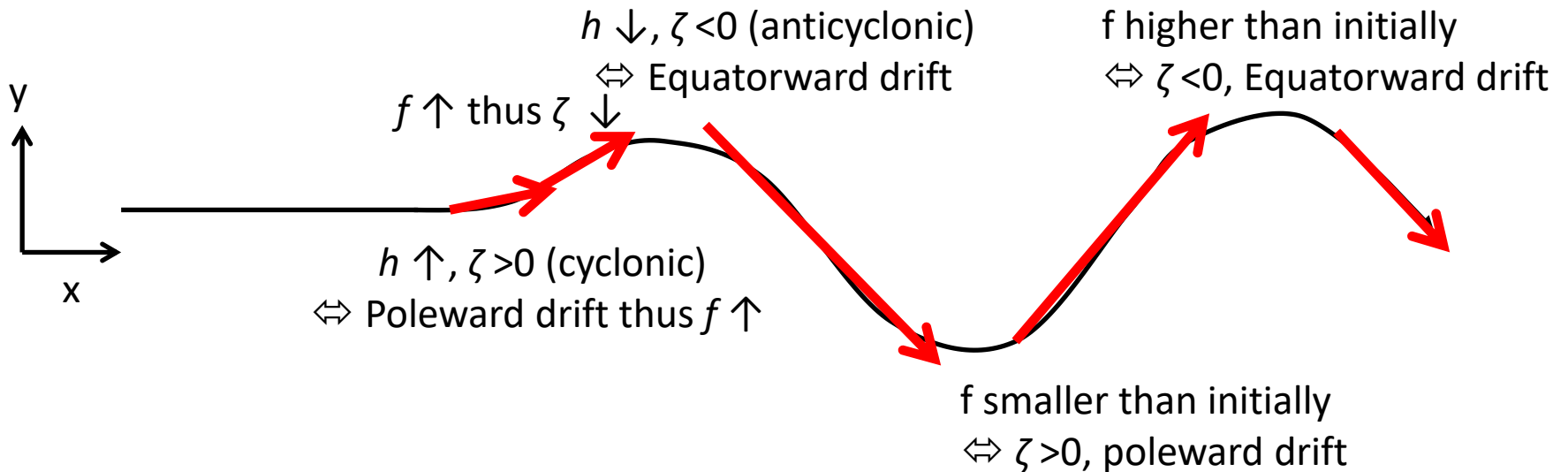
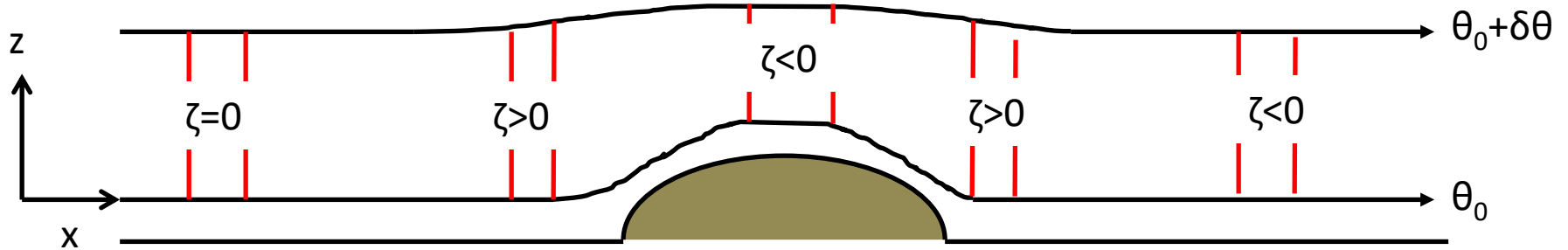


# Potential vorticity conservation: planetary waves (1)

Westerly flow

$$PV = (\zeta_{\theta} + f) \left( -g \frac{\partial \theta}{\partial p} \right) = const$$

= 1/h



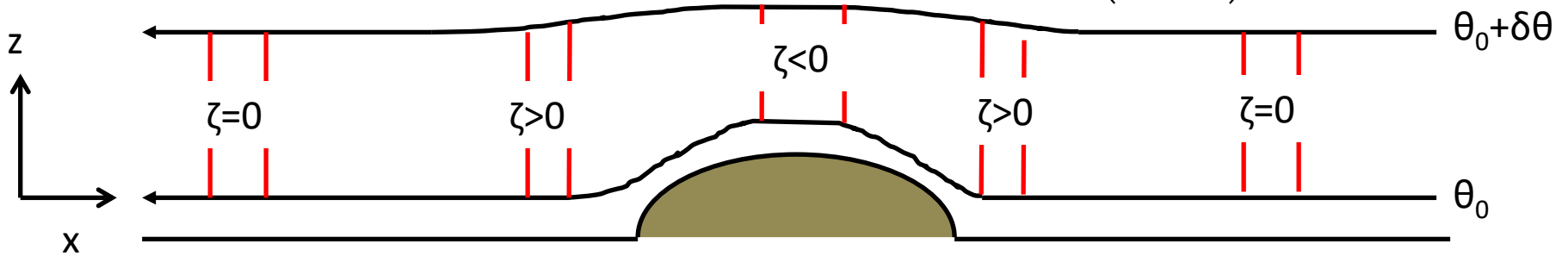
**This mechanism generates planetary waves under westerly circulation**

# Potential vorticity conservation: planetary waves (2)

Easterly flow

$$PV = (\zeta_{\theta} + f) \left( -g \frac{\partial \theta}{\partial p} \right) = const$$

= 1/h



**Return to initial state**

$h \uparrow, \zeta > 0$  (cyclonic)  
 $\Leftrightarrow$  Equatorward drift  $f \downarrow$

$h \uparrow$ : increasing of  
 absolute vorticity  $\Leftrightarrow$   
 cyclonic &  $f \uparrow$

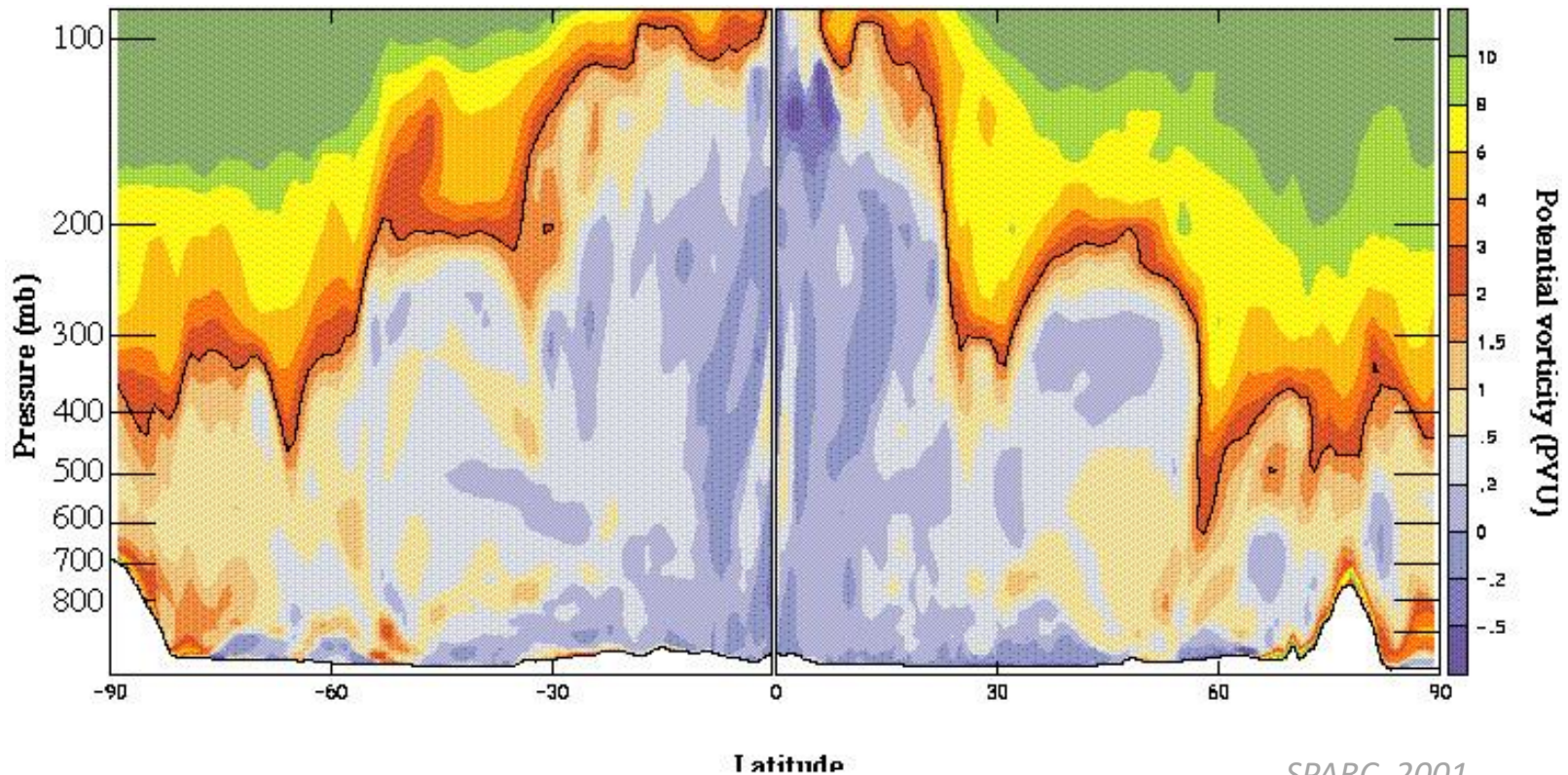
$h \downarrow$ : reduction of  
 absolute vorticity  $\Leftrightarrow$   
 anticyclonic &  $f \downarrow$

**No planetary wave generation under easterly flow**

# Potential vorticity: dynamical tropopause definition

$$PV = (\zeta_{\theta} + f) \left( -g \frac{\partial \theta}{\partial p} \right) = const$$

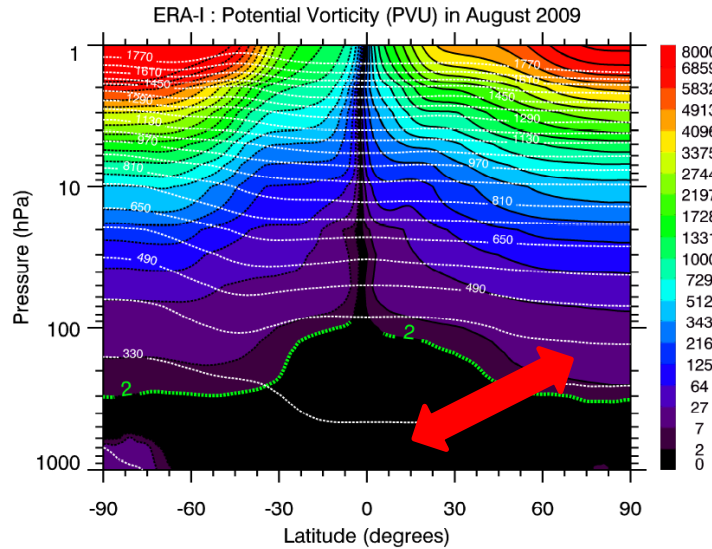
Tropopause definition as PV surface of 2 pvu (1 pvu =  $10^{-6} \text{ K m}^2 \text{ kg}^{-1} \text{ s}^{-1}$ ). Use of the increase statistic stability at the tropopause.



SPARC, 2001

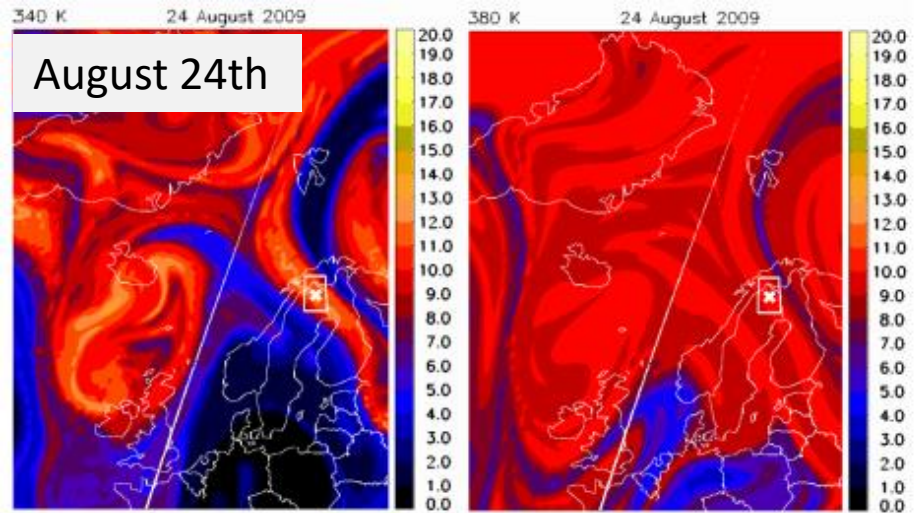
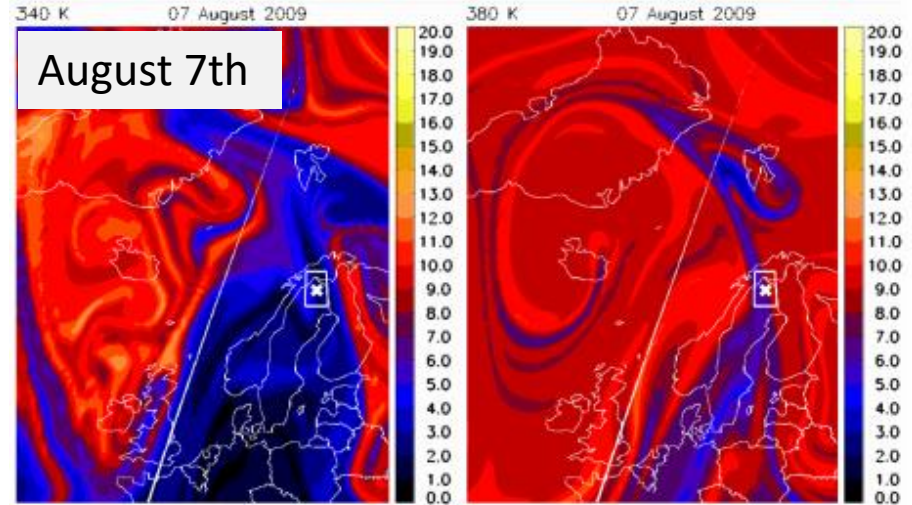
## Stratosphere/troposphere exchanges

### Dynamical conditions (PV)

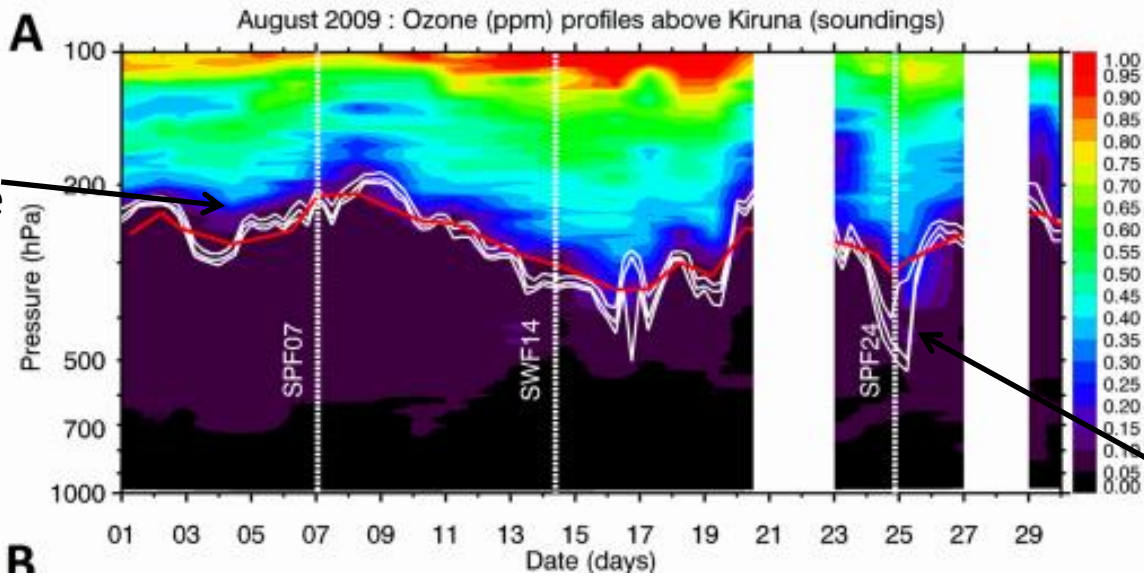


Iisentropic transport

Balloon campaign in polar region

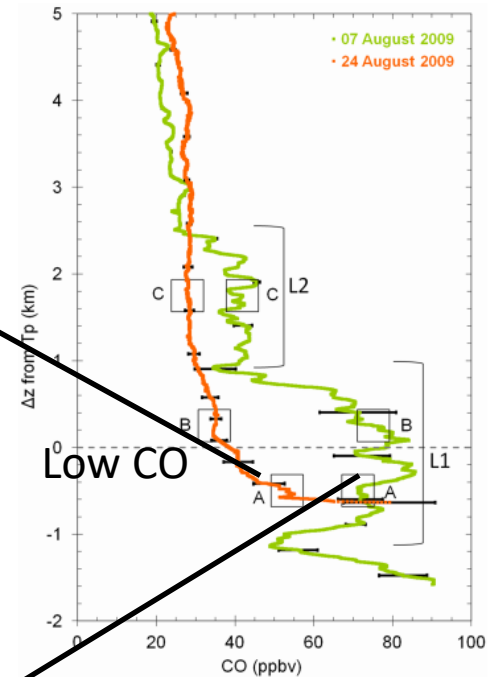


# Stratosphere/troposphere exchanges



Ozone (ppm)

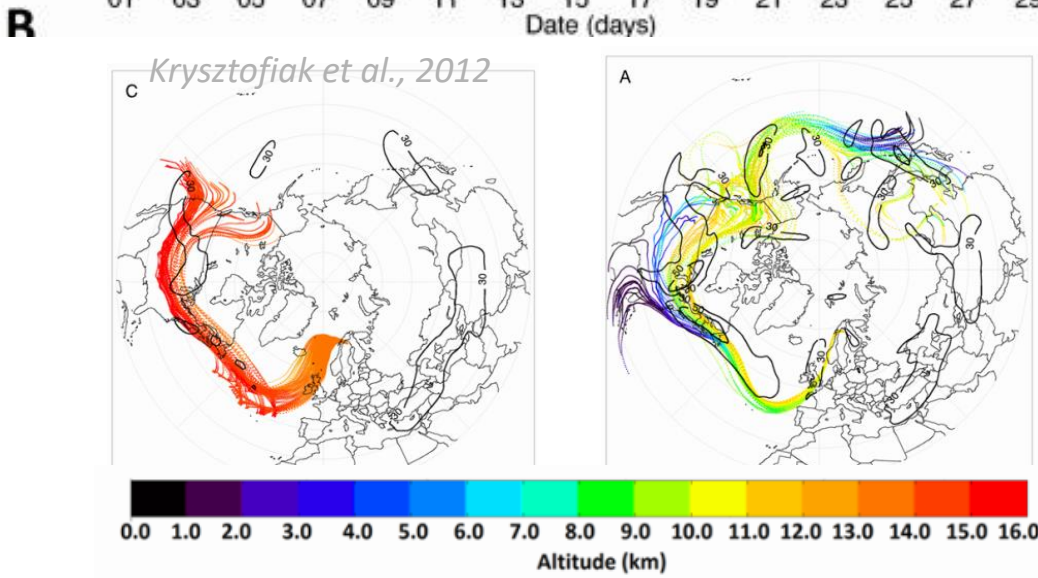
CO profile



Low CO

High CO

(Kiruna, summer 2009)



Dynamical tropopause

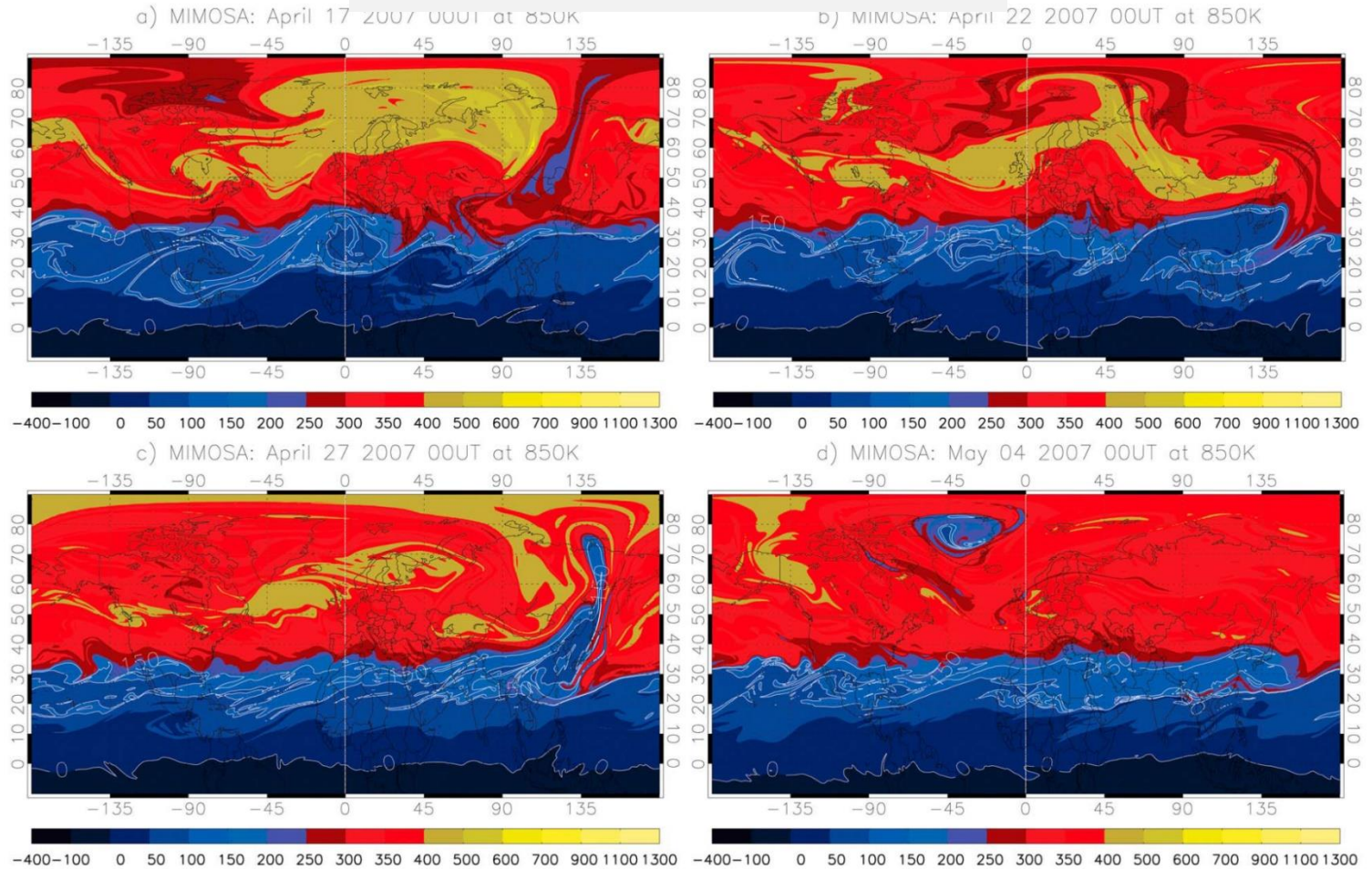
10-days Backward trajectories  
computed from FLEXTRA

Kryztofiak et al. (2012)

# Stratospheric horizontal transport

~30 km

Potential vorticity maps (MIMOSA)



# V. Stratosphere

# Stratospheric circulation

During polar winter

Strong decrease of temperature

Polar VORTEX formation

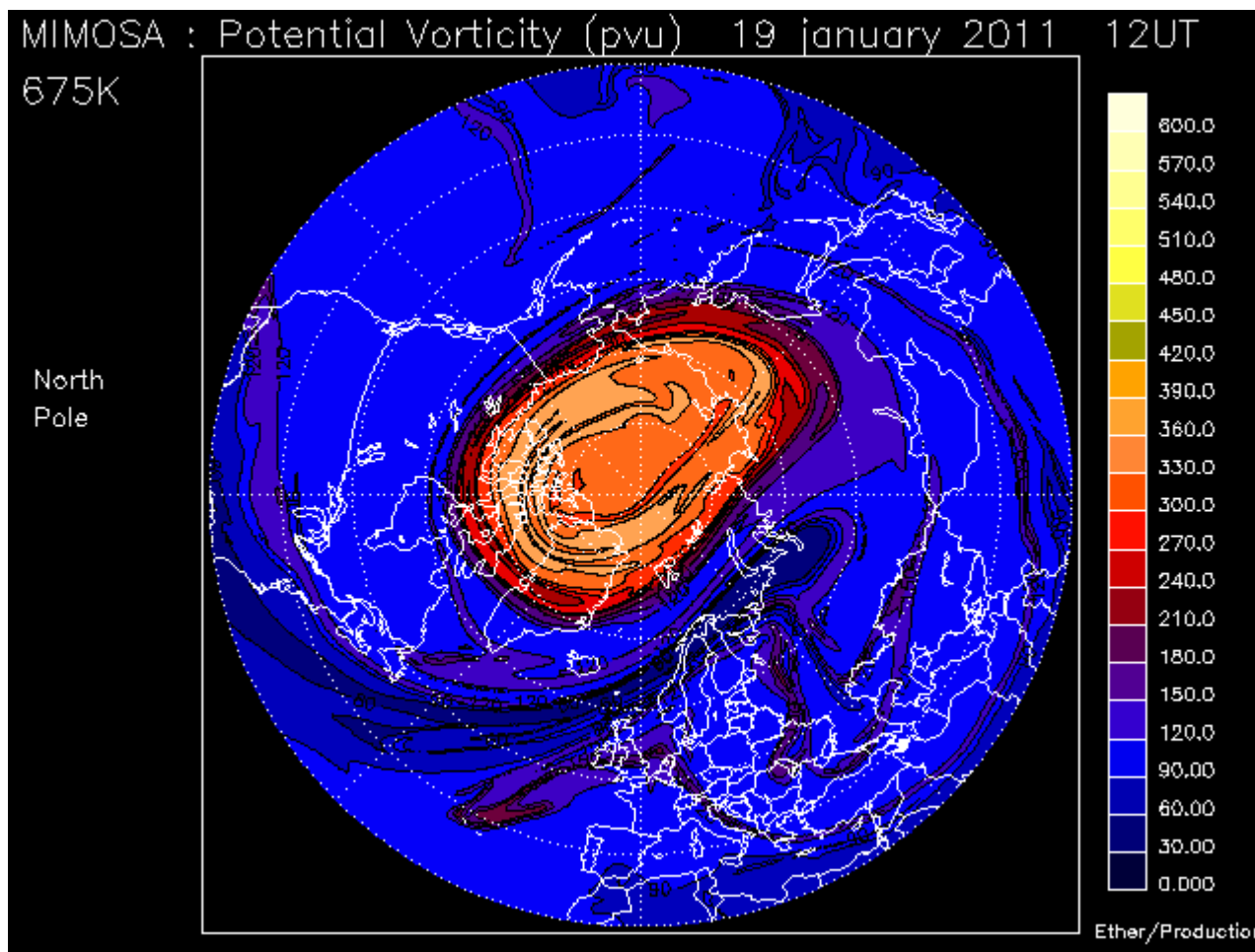
Strong wind  
100 m/s





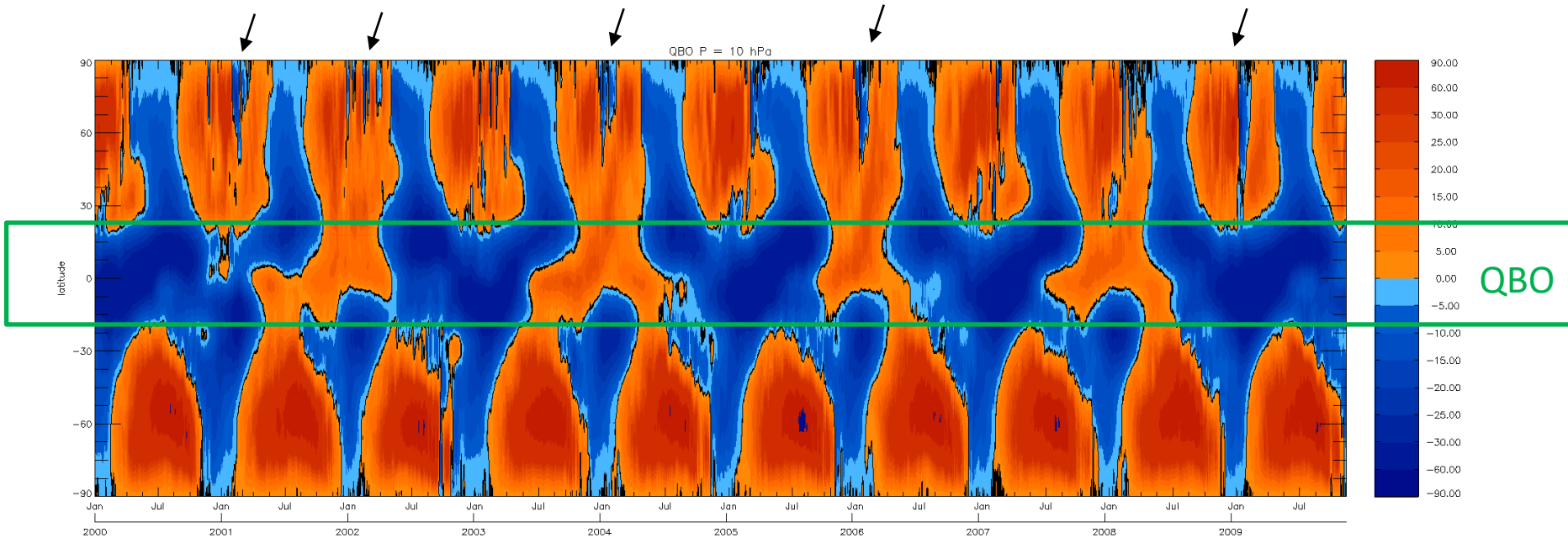
# Polar vortex

- Potential vorticity map Forecasts on AERIS Database



# Zonal circulation cycles at 10 hPa

Sudden Stratospheric Warming → polar vortex displacement or break up



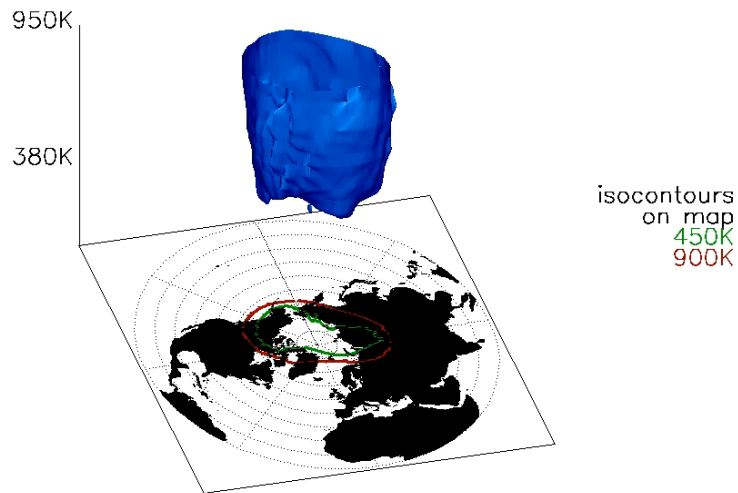
Polar vortex (Esterlies)

Summer polar Anticyclone (Westerlies)

Equatorial Quasi Biennial Oscillation -QBO

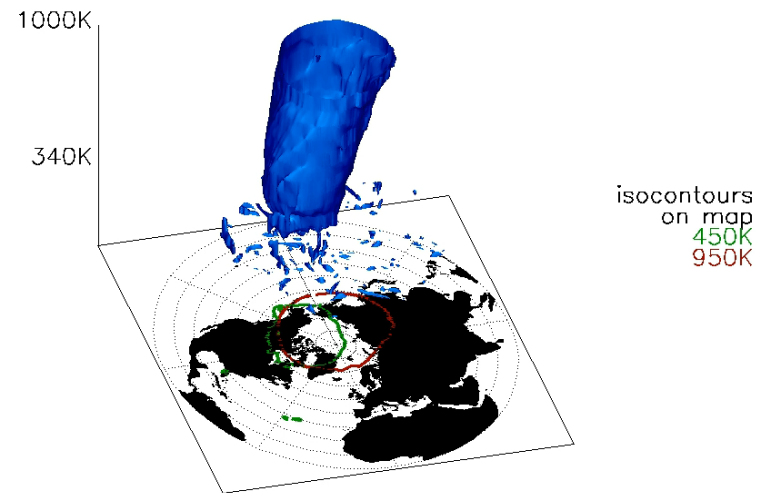
# Polar vortex as a function of altitude

MIMOSA : lait PV normalized with 380K  
01/01/2010 12H



Vortex break-up in two lobes  
-> Sudden Stratospheric Warming

MIMOSA : lait PV normalized with 380K  
01/01/2011 12H

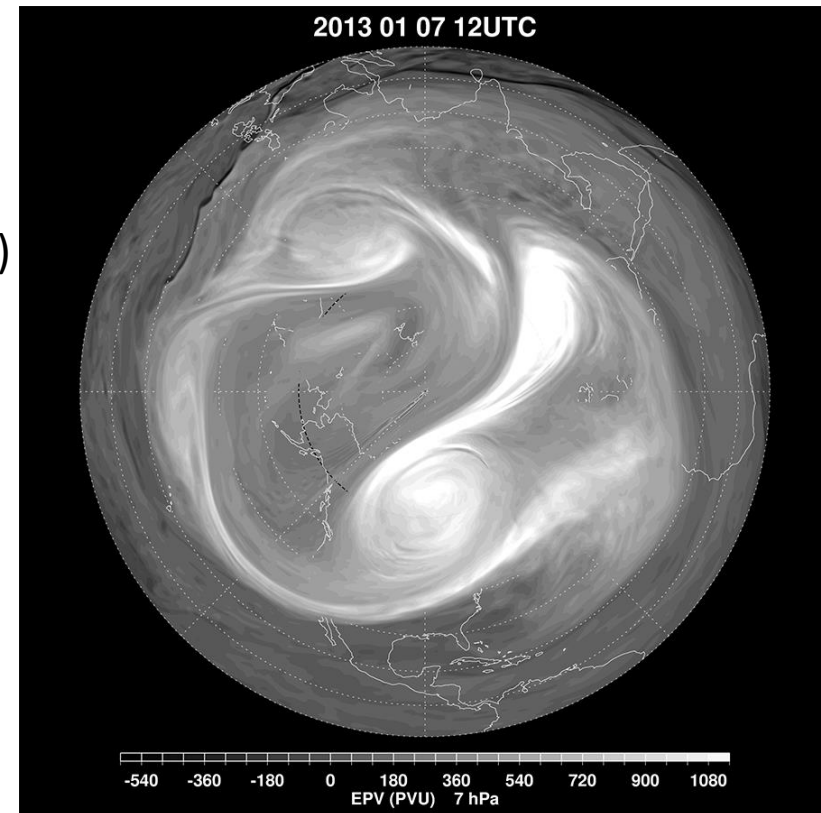


Unusual stable polar vortex  
without major SSW

# Arctic Sudden Warming of January 2013

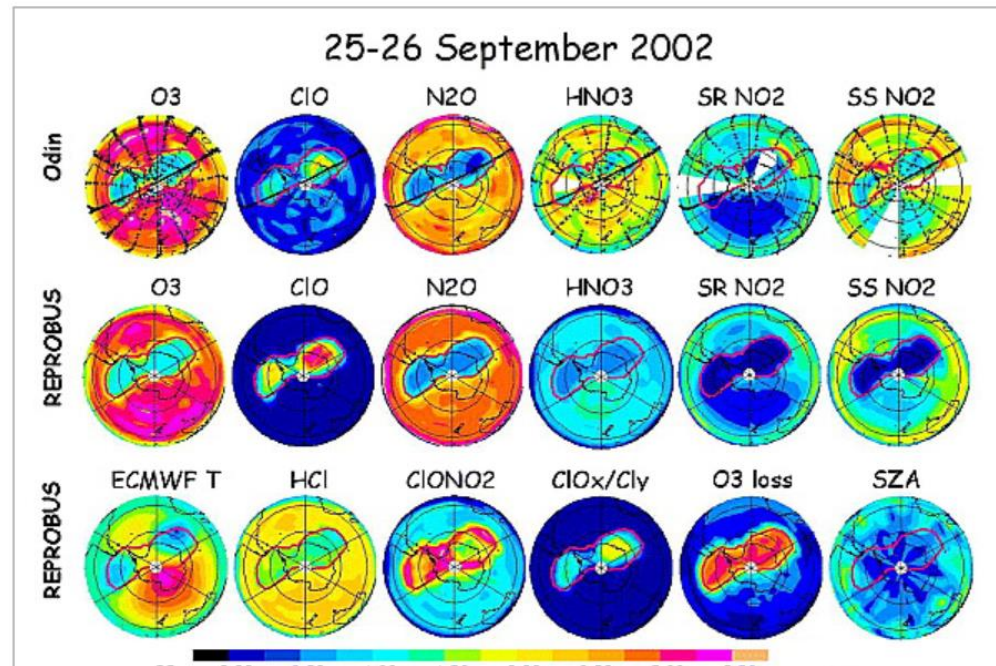
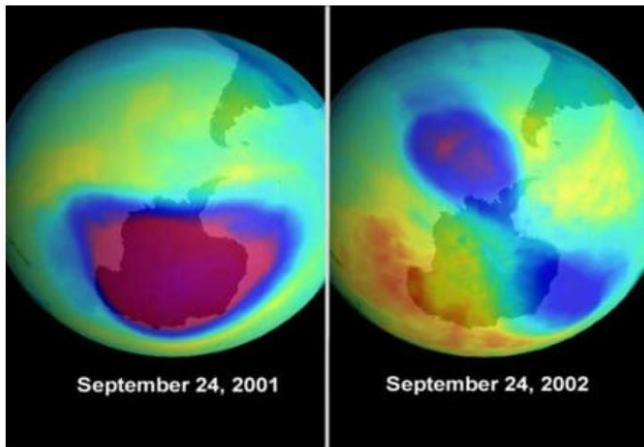
- the vortex distorting and breaking up, coincident with the increase in polar temperatures at 10 hPa.
- By January 7, 2013, the shearing of the original polar vortex by the winds led to the existence of three smaller, interconnected vortices (over Canada, northern Eurasia, northeastern Siberia..)

**Lawrence Coy, Steven Pawson**



# Antarctic polar vortex Split in 2002

In September 2002 the Antarctic polar vortex split in two under the influence of a sudden warming



*Polar vortex evolution during the 2002 Antarctic major warming as observed*

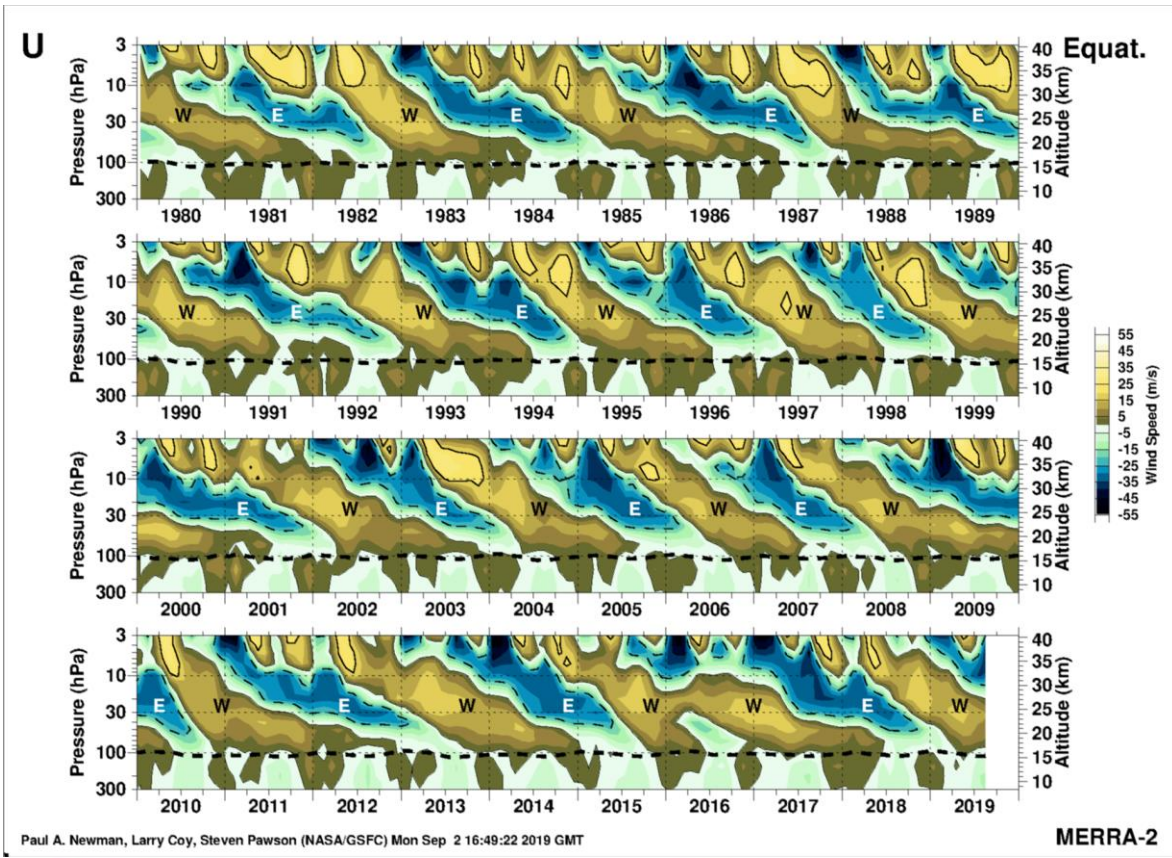
*by the Odin satellite*

*P. Ricaud et al. 2005, JGR, [doi.org/10.1029/2004JD005018](https://doi.org/10.1029/2004JD005018)*

## Stratosphere and Quasi Biennial Oscillation in equatorial region

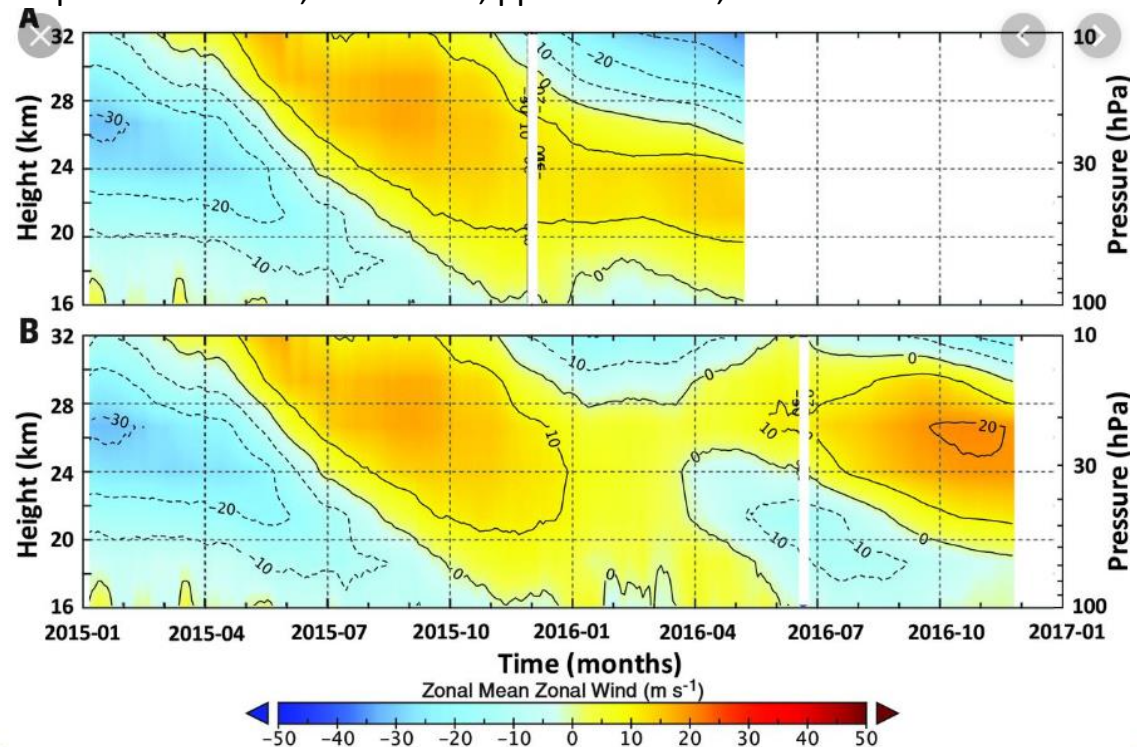
- One of the most repeatable phenomena seen in the atmosphere, the quasi-biennial oscillation (QBO) between prevailing eastward and westward wind jets in the equatorial stratosphere (approximately 16 to 50 kilometers altitude)

Mean Zonal wind



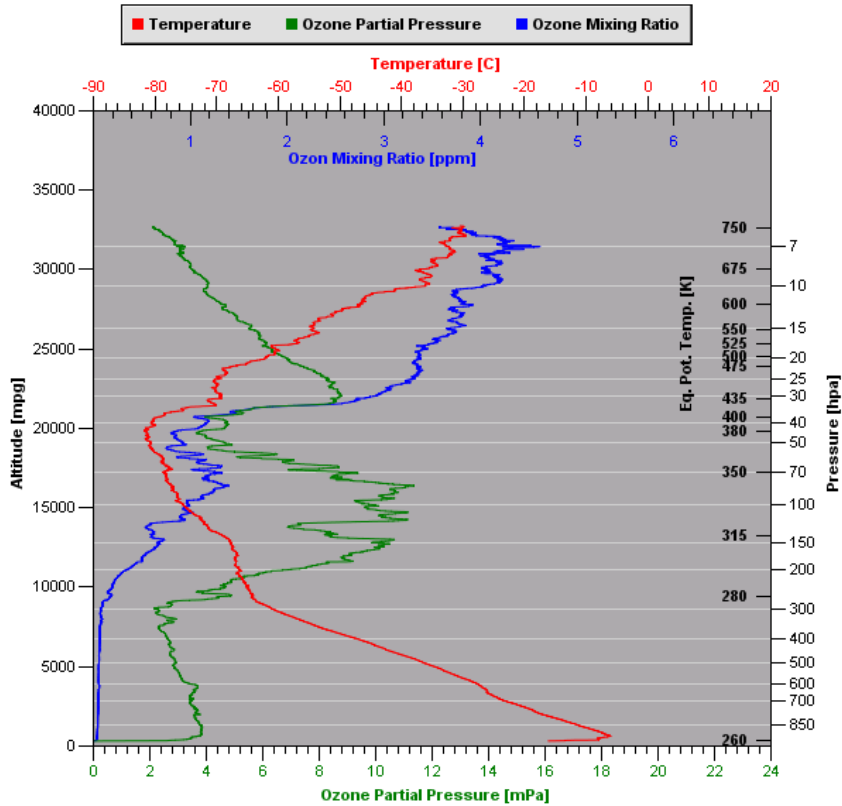
# An unexpected disruption of the atmospheric quasi-biennial oscillation 2016

- QBO was unexpectedly disrupted in February 2016. An unprecedented westward jet formed within the eastward phase in the lower stratosphere and cannot be accounted for by the standard QBO paradigm based on vertical momentum transport.
- *S. M. Osprey et al., Science* 23 Sep 2016: Vol. 353, Issue 6306, pp. 1424-1427, DOI: 10.1126/science.aah4156

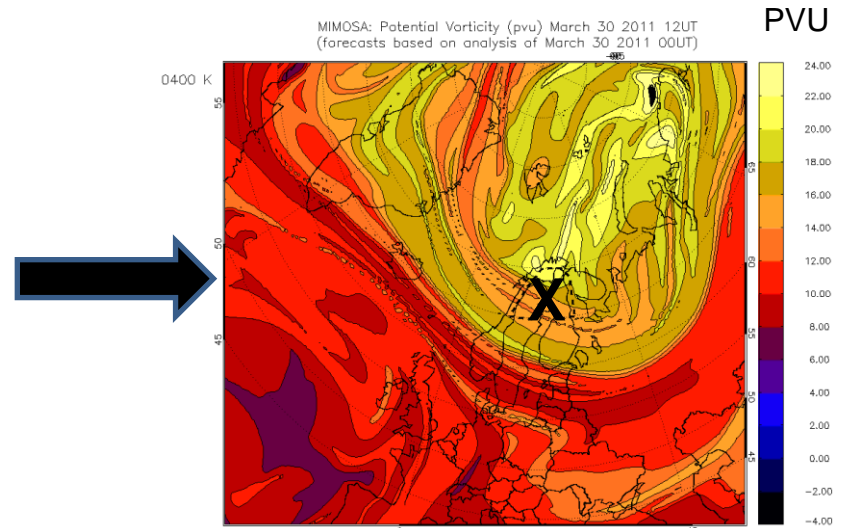


# Ozone loss : winter2010/spring 2011 Event

## Temperature and Ozone profiles



## Potential vorticity map 400K isentropic surface (~65 hPa, ~19 km)



MIMOSA model (Hauchecorne et al. 2002)

Geophysical conditions on isentropic surface 400K inside the vortex

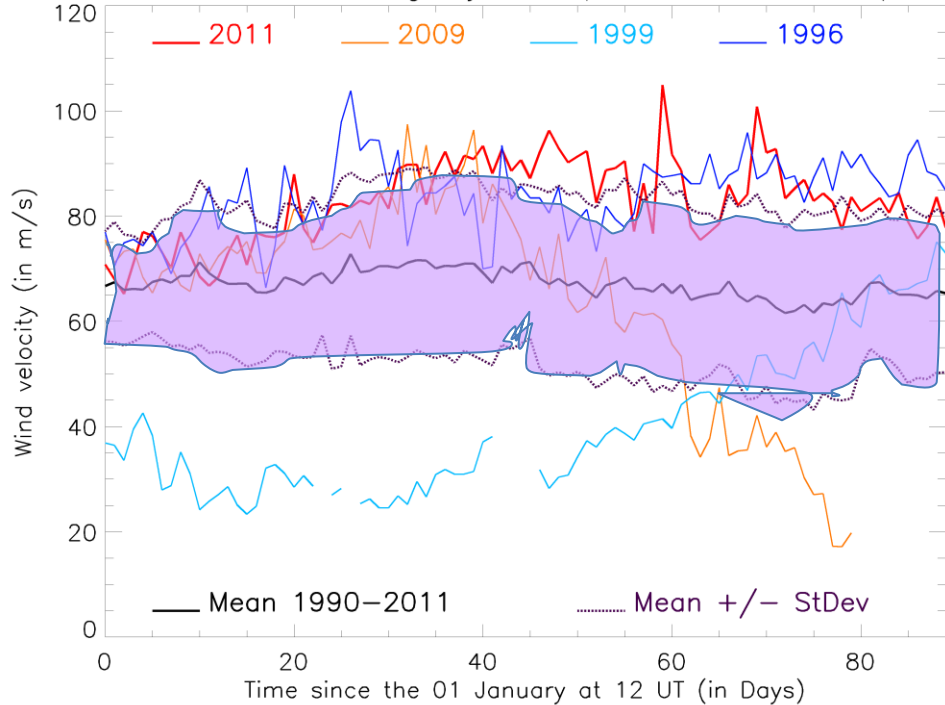
- Temperature < - 85°C
- Ozone mixing ratio < 1 ppmv, ~ [17;22] km



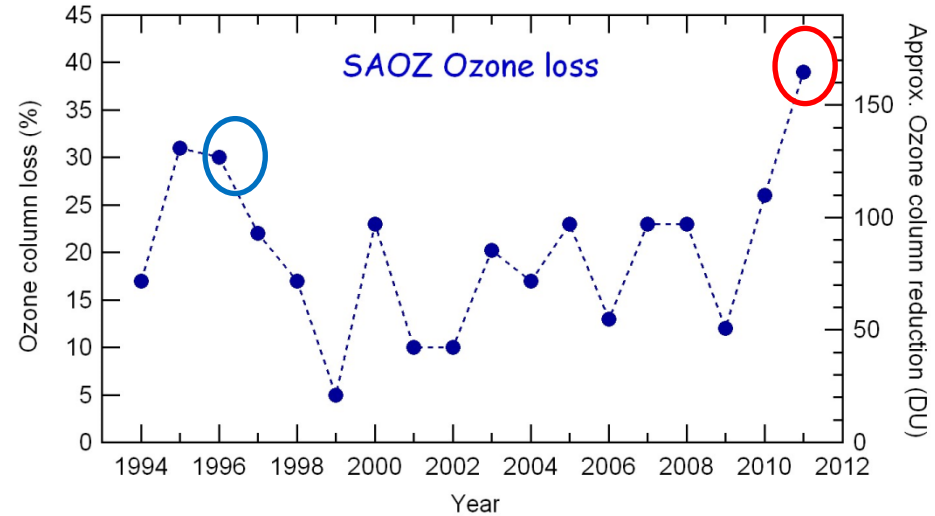
# 2011 Very intense polar vortex in Arctic region



ERA-INTERIM : Polar night jet comparisons at 850 K ( $\sim 31$ km)



## Ozone loss never seen during 20 last years



<http://saoz.obs.uvsq.fr/O3Loss.html>,  
 Florence Goutail, LATMOS.

Exceptional conditions for dynamical and chemical studies

# 2011 unprecedented Arctic ozone loss

- Unprecedented Arctic ozone loss in 2011

Gloria L. Manney, Michelle L. Santee, Markus Rex, Nathaniel J. Livesey, Michael C. Pitts, Pepijn Veefkind, Eric R. Nash, Ingo Wohltmann, Ralph Lehmann, Lucien Froidevaux, Lamont R. Poole, Mark R. Schoeberl, David P. Haffner, Jonathan Davies, Valery Dorokhov, Hartwig Gernandt, Bryan Johnson, Rigel Kivi, Esko Kyrö, Niels Larsen, Pieter F. Levelt, Alexander Makshtas, C. Thomas McElroy, Hideaki Nakajima, Maria Concepción Parrondo *et al.*

*Nature* (2011) doi:10.1038/nature10556

Received 03 May 2011 Accepted 07 September 2011 Published online 02 October 2011

- The Arctic vortex in March 2011: a dynamical perspective

M. M. Hurwitz, P. A. Newman, and C. I. Garfinkel

*ACPD* (2011)

Received: 5 July 2011 – Accepted: 2 August 2011 – Published: 5 August 2011

# What's new ? winter 2019/spring 2020

Atmos. Chem. Phys., 21, 14019–14037, 2021  
 https://doi.org/10.5194/acp-21-14019-2021  
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 the Creative Commons Attribution 4.0 License.



Atmospheric  
 Chemistry  
 and Physics  
 Open Access  
 EGU

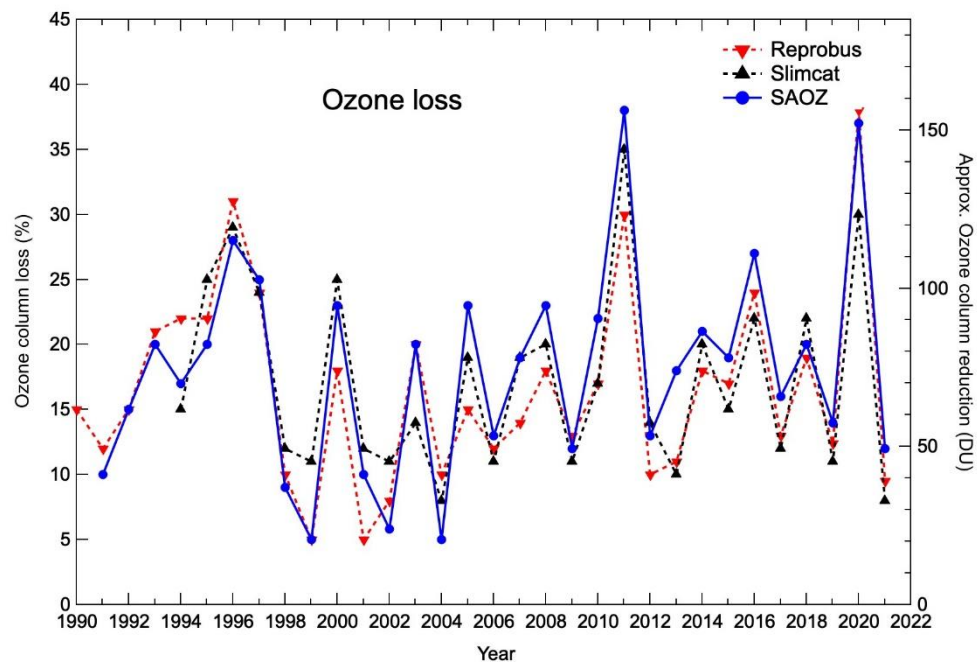
## Exceptional loss in ozone in the Arctic winter/spring of 2019/2020

Jayanarayanan Kuttippurath<sup>1</sup>, Wuhu Feng<sup>2,3</sup>, Rolf Müller<sup>4</sup>, Pankaj Kumar<sup>1</sup>, Sarath Raj<sup>1</sup>,  
 Gopalakrishna Pillai Gopikrishnan<sup>1</sup>, and Raina Roy<sup>5</sup>

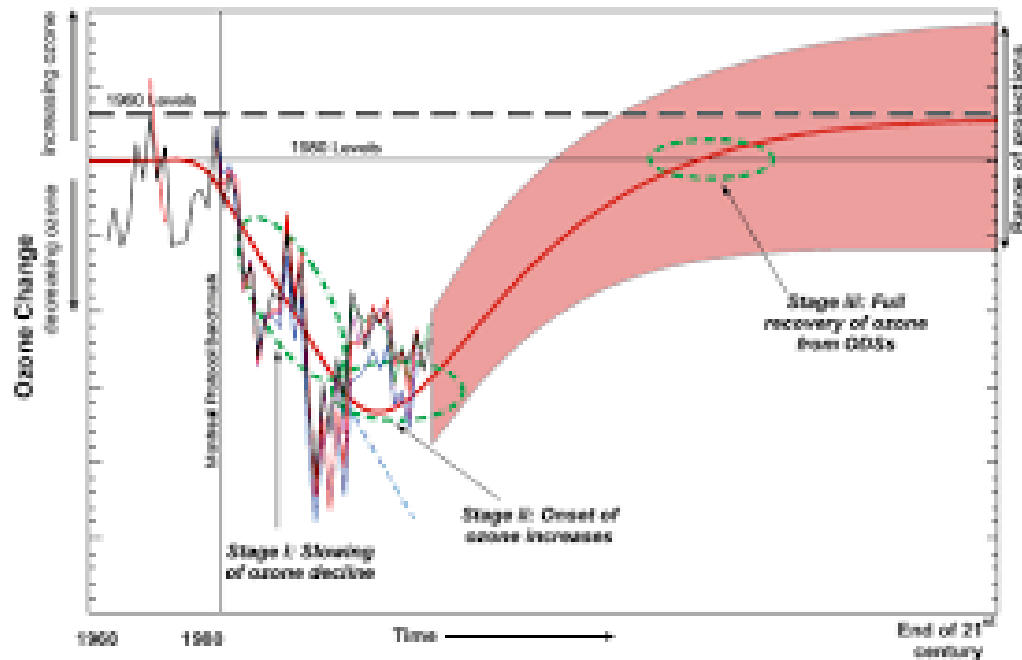
F. Goutail, LATMOS/CNRS

<http://saoz.obs.uvsq.fr/O3Loss.html>

- 2020 ozone loss similar to 2011 ozone loss :  
 -38% of the O<sub>3</sub> total column  
 -150 O<sub>3</sub> DU

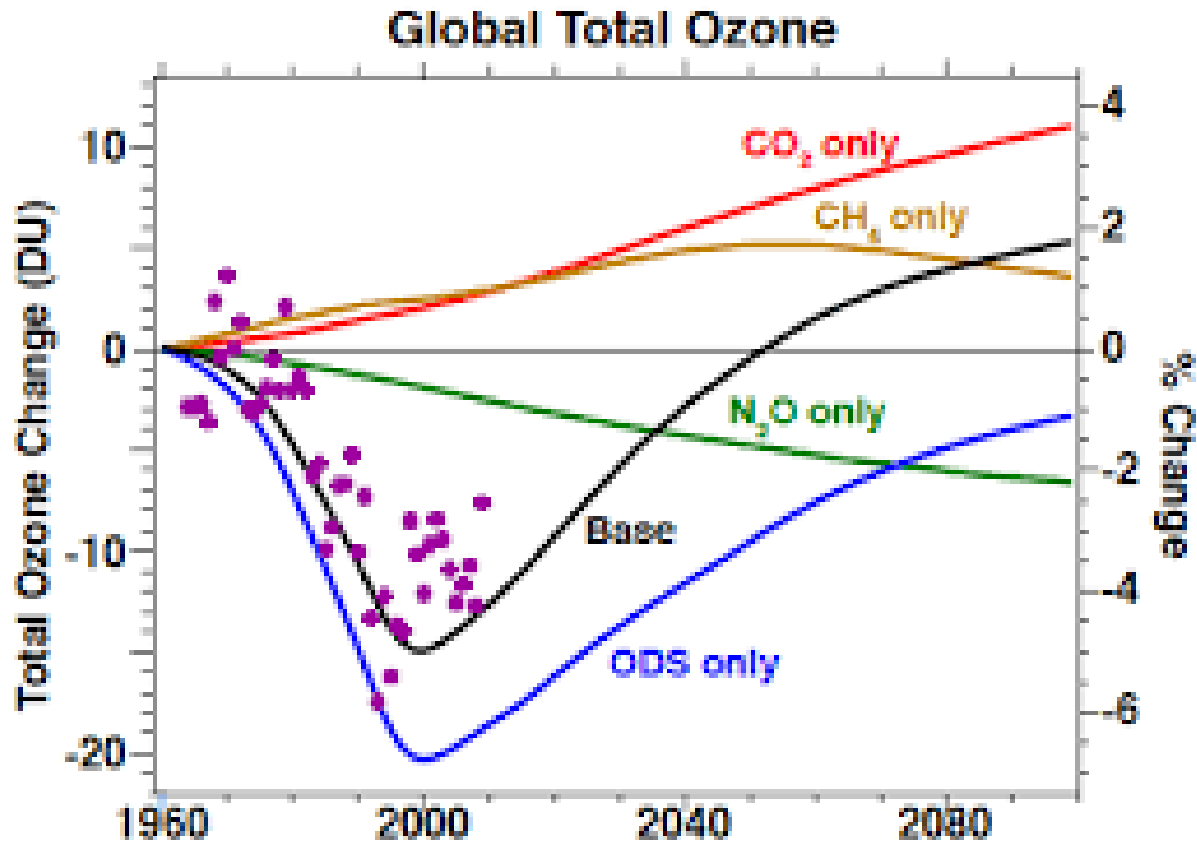


# Ozone layer recovery



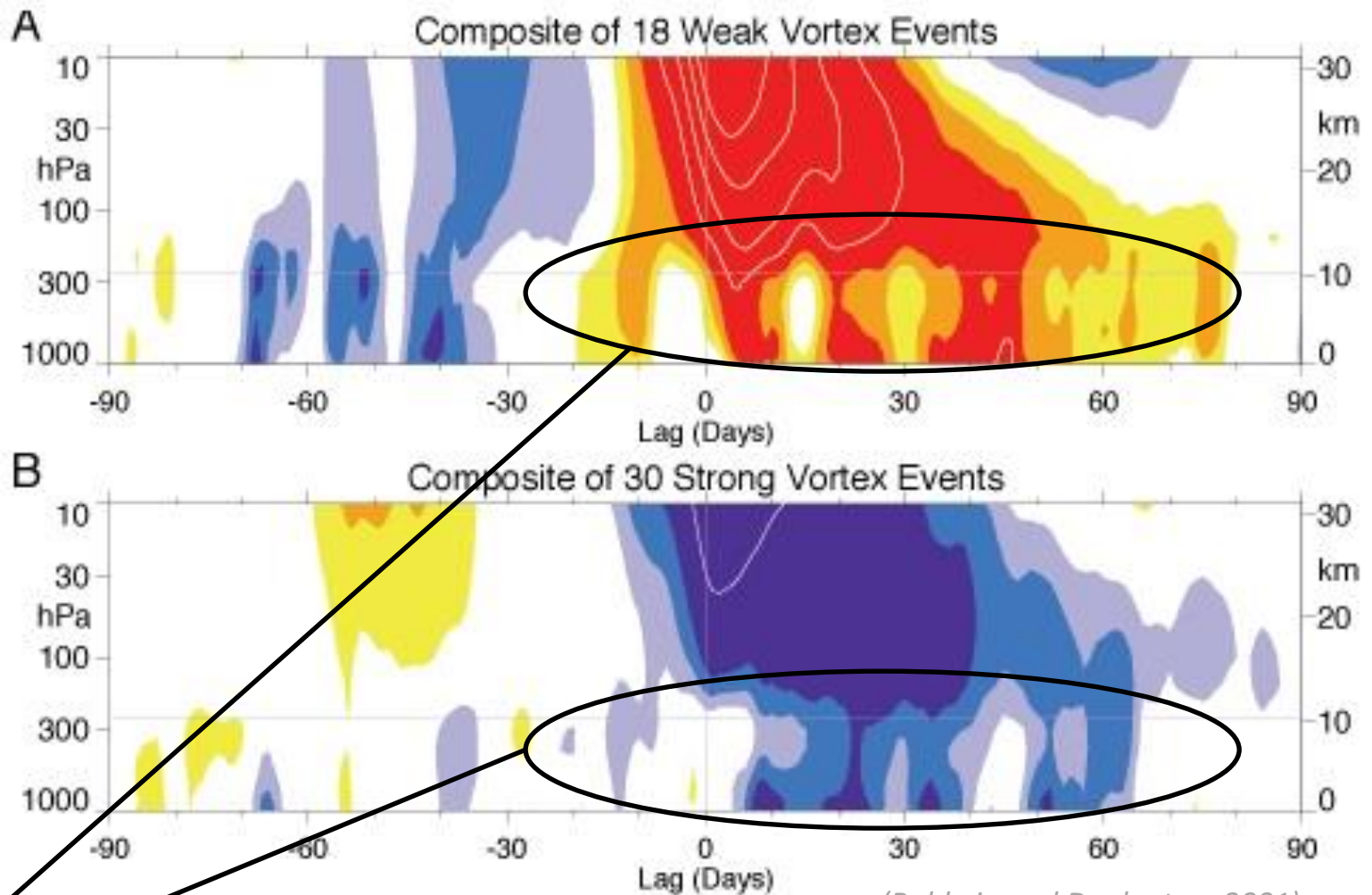
- Recovery could be possible mis 21th century but depends on GES in the troposphere

# Ozone layer recovery



- Dépend climate forcing due to impact of GES on

# Stratospheric influence on surface climate



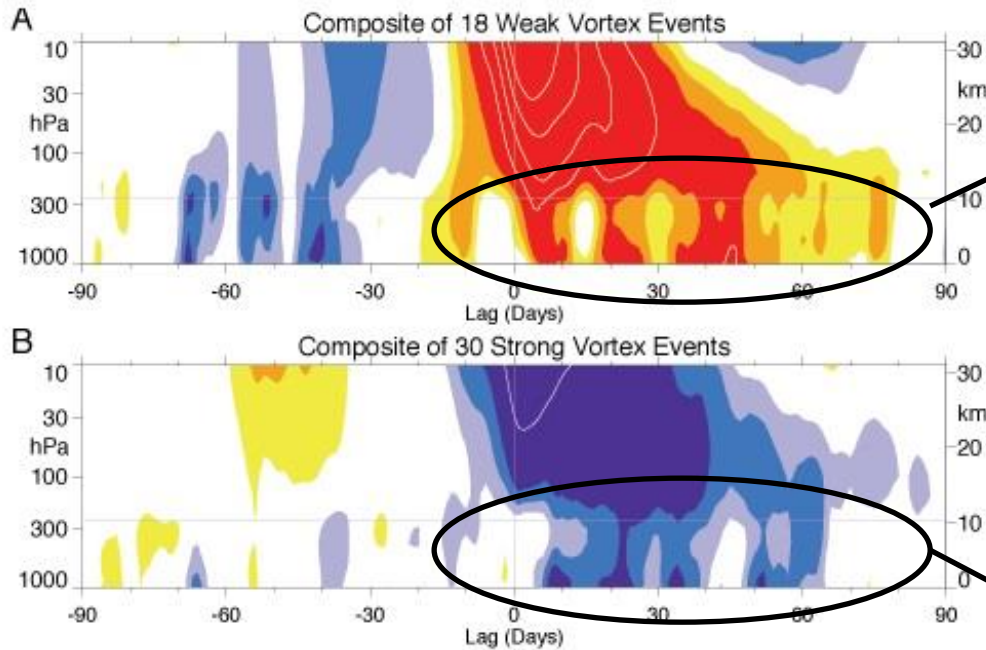
(Baldwin and Dunkerton, 2001)

**Polar vortex variability affects the surface !**

What does it mean ?

# Surface response

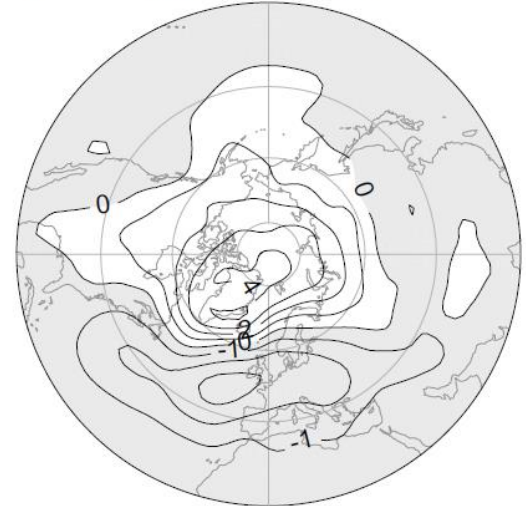
Downward propagation of the signal



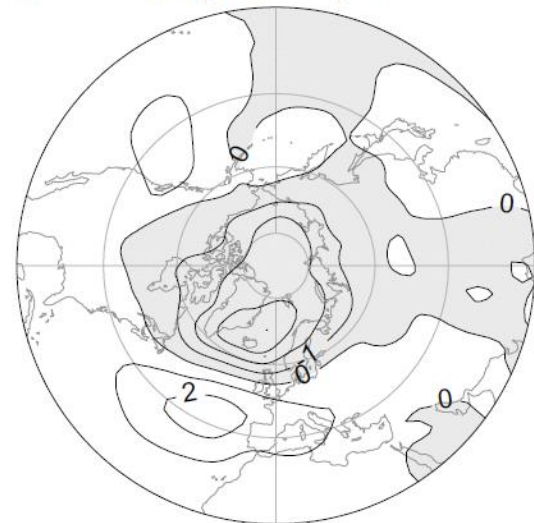
*(Baldwin and Dunkerton, 2001)*

## Surface response (Sea Level Pressure)

A Weak Vortex Regimes



B Strong Vortex Regimes



The signal project onto a „North Atlantic Signal“  
 Influence on troposphere weather ?

## Take home messages

- Atmosphere circulation: redistribution of energy due to SW absorption and LW emission asymetry
- Stratosphere radiatively balanced but not the troposphere  $\Leftrightarrow$  latent heat release from water vapour crucial for troposphere energy budget and circulation.
- Negative vertical temperature gradient in the troposphere leads to high potential unstability and convection, while the stratosphere is very stable with very slow vertical transport.
- Mid-latitudes regional circulation strongly depends on Coriolis acceleration
- Brewer-Dobson large-scale circulation in the stratosphere mainly driven by Rossby waves which propagate from the troposphere and which are responsible for the stratospheric dynamical variability
- Troposphere weather coupled with stratosphere dynamical conditions (but not only !)
- Chemical and dynamical processes are strongly interacting





**ESPRI Data Centre** (former Ether)

ESPRI is a data centre of the French Atmosphere Infrastructure AERIS

- AERIS data center

Model Forecasts :

3D CTM REPROBUS

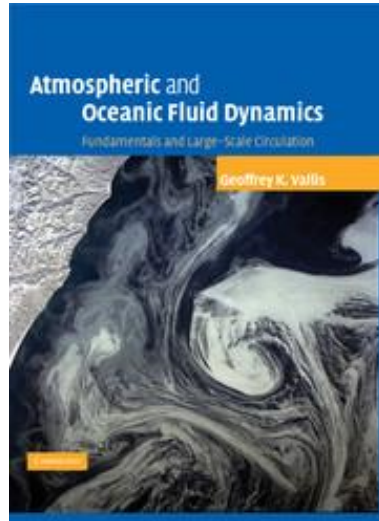
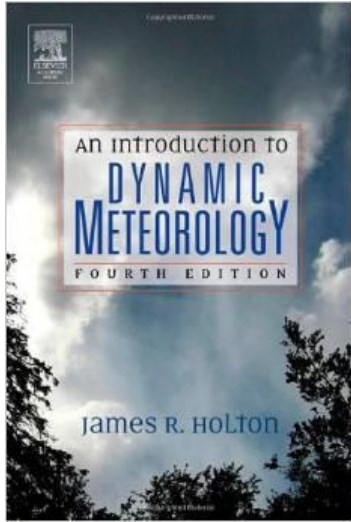
Reactive Processes Ruling the Ozone Budget in the Stratosphere) temporal evolution of 55 stratospheric species by 147 réactions chimiques [Lefevre et al., 1998].

MIMOSA :

potential vorticity contour advection model (hauchecorne et al. 2001)

- <http://espri.aeris-data.fr>

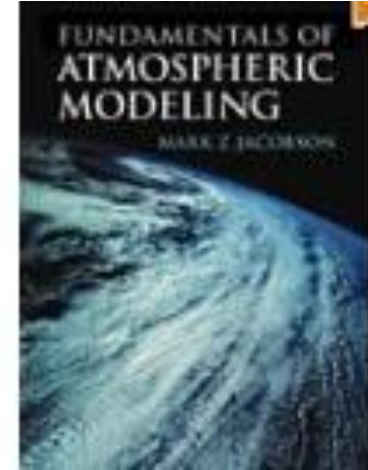
## Some literature



### MIDDLE ATMOSPHERE DYNAMICS



• David G. Andrews • James R. Holton •  
• Conway B. Leovy •



Physique  
et chimie  
de l'atmosphère

Avec CD-ROM

Belin

